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EXAMINATION OF PRE-SERVICE MATHEMATICS TEACHERS' KNOWLEDGE OF TEACHING FUNCTION CONCEPT[1]

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Abstract: Teaching of mathematics could be improved with teachers who have a strong mathematical knowledge and have an ability to reflect this knowledge on their teaching. Therefore, it is important to develop mathematics teachers' theoretical and pedagogical knowledge. This study was designed to examine pre-service secondary mathematics teachers' (PSMT) mathematical knowledge for teaching (MKT) in terms of function concept. In order to examine PSMT's MKT regarding function concept, case study design was used in the study. Three PSMTs', who were at the fifth year in the program, were selected as the participants. The data was collected by observing and recording PSMTs' teaching of function concept as well as by examining their lesson plan of the concept. Descriptive and content analysis were used to analyze the collected data. The results revealed that PSMTs' had limited knowledge regarding teaching of function concept. Particularly, they had difficulties to reflect their knowledge of function concept on their teaching. The results also showed that experience is directly related to teaching of function concept, time and classroom management and communication with students. Therefore, it is important to improve both in-service and pre-service teachers' MKT by considering the effect of experience and abilities in teaching.

Key words: teaching function concept, mathematical knowledge for teaching (MKT), pre-service mathematics teachers' education

1. Introduction

Existing standards has focused on the importance of teachers in mathematics teaching (e.g. National Council of Teachers of Mathematics [NCTM], 2000), and effective mathematics teaching is possible with mathematics teachers who have a strong mathematical knowledge and have an ability to reflect this knowledge to their teaching. Ball and Bass (2002) indicated that improving students learning of mathematics depends on developing their teachers' knowledge. Many researchers have interested in the knowledge types that teachers must have for years. For instance, Shulman (1986) was eager to learn, "How do teachers decide what to teach, how to represent it, how to question students about it, and how to deal with problem of misunderstanding?" (p. 8). Therefore, Shulman and his colleagues constructed a theoretical framework called as pedagogical content knowledge (PCK). By taking Shulman's work as a base and an example, many researchers defined new theoretical frameworks such as Technological Pedagogical Content Knowledge (TPACK) (Mishra and Koehler, 2006) and Mathematics Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008). Different from other studies, Ball, Thames, and Phelp (2008) focused on the teachers' knowledge which is specific to mathematics education, and constructed MKT framework which is widely used in this area.

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Particularly, Ball and her colleagues (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005; Thames & Ball, 2010) eloborated Shulman's notions of subject matter content knowledge (SMCK) and PCK, and defined and developed the MKT framework.

Hill, Rowan and Ball (2005) defined MKT as "the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). They stated that the 'work of teaching' includes explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, and proofs. Then, Ball et al. (2008) focused on 'the work of teaching', and they noticed that MKT and teaching skills were different types of knowledge. MKT covers all the knowledge type that is neccessary for effective teaching since mathematics teachers belong to the special class among mathematics users and they are ahead of professional mathematics users (Steel, Hillen, & Smith, 2013). There are significant results that show the importance of MKT framework in order to connect the effect of teachers' mathematics teaching on students obtained mathematical knowledge (Baumert et al. 2010; Charalambous 2010; Hill, Rowan, & Ball, 2005; Tchoshanov 2011).

The concept of function is a significant and unifying concept of modern mathematics (NCTM, 2000; Selden & Selden, 1992; Yerushalmy & Schwarz, 1993). Particularly, learning functions is essential in secondary mathematics in which teaching of it starts in 9th grade and continues to college levels in Turkey. In addition to that, learning the concept of function is required to learn many concepts in other disciplines which is related to the field of applied mathematics (Ronau, Meyer, Crites & Dougherty, 2014). While the concept of function is such important, many researchers indicated that students have difficultiy in learning and understanding the concept (Sajka, 2003; Sierpinska, 1992). Existing studies illuminates the reason of these difficulties, and Kieran (1992) focused on the role of teaching function concept in students' learning. In other words, he illuminated teachers' role in students' learning of the concept. From this point of view, it was thought that mathematics teachers' knowledge regarding the concept of function needs to seek in order to develop the quality of teachers' knowledge of mathematics teaching and students' learning of the concept. Therefore, the main goal of this study was to examine pre-service secondary mathematics teachers' (PSMT) teaching of the concept of function within the MKT framework.

2. Theoretical Framework

Ball and her colleagues (Ball, Hill, & Bass, 2005; Ball, Thames, & Phelps, 2008; Hill, Rowan, & Ball, 2005; Thames & Ball, 2010) eloborated Shulman's notions of SMCK and PCK, and defined and developed the mathematicak knowledge for teaching (MKT) framework. Based on their analysis of the mathematical demands for teaching, they suggested that Shulman's (1986) content knowledge (CK) could be subdivided into common content knowledge (CCK), specialized content knowledge (SCK), and that his PCK could be divided into knowledge of content and students (KCS), and knowledge of content and teaching (KCT). They further suggested that MKT framework includes all these domains, and horizon content knowledge (HCK) and knowledge of content and curriculum (KCC) (See Figure 1).

Ball et al.'s (2008) definitions of the domains could suggest that MKT framework could be interpreted as elaborating on the construct of PCK. The CCK was defined as "Mathematical knowledge and skill used in settings other than teaching." (Ball et al., 2008, p. 399). In this context, teachers should know how to use the related material in their teaching, recognize students' misconceptions or textbook's mistakes, need to use the terms and notation correctly, and be able to do the work that they assign students. The SCK was defined as "the mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p. 400). Teachers, who have SCK, are able to do a kind of mathematical work and operations that others cannot. The definitions of KCS and KCT directly coincide with Shulman's notion of PCK. According to Ball and her colleagues, teachers with KCS have the ability to choose an appropriate example that students find interesting and motivating, to assign a task that students anticipate with an understanding if they will find it easy or hard and to interpret students emerging and incomplete thinking as expressed in the ways that students use language. Going further, Ball et al. (2008) stated that during classroom discussions, teachers with KCT are able to decide when to pause for more clarification, when to use a student's remark to make a mathematical point, and when to ask a new question or pose a new task to further students' learning (p. 401). HCK is defined as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al., 2008, p.403). In this context, teachers should know how different grades mathematics curriculum is related to each other. KCC is directly taken from Shulman's (1986) work; it is defined as "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances" (p. 10).

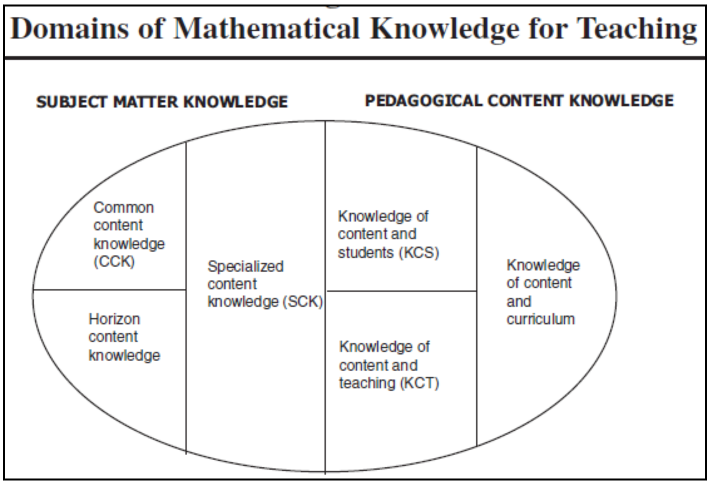

Figure 1. *Domains of MKT framework (Adapted from Ball et al., 2008)*

In this study, we wondered about what specific knowledge teachers should have in order to help students in developing rich and connected concept of function. We believed that identifying PSMTs' CCK, SCK, KCS and KCT could be helpful to answer the question. Additionally, we thought that since the participants of this study were PSMTs and they did not have any experience and a chance to teach in a real class environment, we only focused on the combination of CCK, SCK, KCS and KCT as the framework of this study (See Table 1).

While the MKT framework was eloborated in this study, Ball et al. (2008), Nyikahadzoyi (2015), and Steel et al.'s (2013) studies were considered. After Ball and her collegues developed the MKT framework, Nyikahadzoyi (2015) and Steel et al. (2013) adapted and developed the MKT framework by specializing in the concept of function. Nyikahadzoyi (2013) and Steel et al.'s (2013) eloborated Ball and her collegues ideas regarding the MKT framework to identify the components of what constitutes teachers' knowledge for teaching of the concept of function. For instance, Nyikahadzoyi (2015) indicated that teachers, who have SCK, should know the different definition and representations of a function, be able to choose the neccessary representation for particular purposes and concepts, have a great collection of neat examples, know the importance of the topic in relation to the whole subject and be able to explain this to the students, and conceive a function as a procedure, process, object, or procept (s. 268). While Steele et al. (2013) defined the components of SCK, they also focused on the importance of definition, using multiple representation and being able to felxibly move them. In this study, Table 1 was constructed as the theoretical framework of this study by considering these three studies. In Table 1, italic bulletins/sentences were adapted from Ball et al.'s study, the bold bulletins/sentences were adapted from Steel et al.'s (2013) study and others were adapted from Nyikahadzoyi's (2013) study. Considering these three studies in the context of SCK that teachers should have, we re-designed the SCK as given in Table 1. Other three knowledge domains (CCK, KCS and KCT) also re-designed by following this way.

Table 1. *Mathematical Knowledge for Teaching Framework Specializing in Function Concept*

Knowledge Domain		Components of The Knowledge Domain
SUBJECT MATTER KNOWLEDGE	Common Content (Function Concept) Knowledge	• know central definitions and properties of functions • know connections between the concept and other mathematical concepts • know relevant applications of functions in and outside mathematical contexts • can successfully complete secondary school students' problems involving the concept and identify incorrect answers or inaccurate definitions of the concept of a function • *use terms and notation correctly* • *know the material they teach*
	Specialized Content (Function Concept) Knowledge	• **evaluate function definitions and consider their utility for teaching** • **(a)** choose different representations for particular purposes and **(b) move flexibly between representations of functions** • **(a)** have a great collection of neat examples; **(b)** know the importance of the topic in relation to the whole subject and **(c)** are able to explain this to students • can conceive a function as a procedure, process, object, or procept • *use mathematical language explicitly* • *explain and justify one's mathematical ideas*
PEDAGOGICAL CONTENT KNOWLEDGE	Knowledge of Content (Function Concept) and Students	• *anticipate what students are likely to think and what they will find confusing* • *predict what students will find interesting and motivating when choosing an example* • ***(a)** anticipate what students are likely to do with it and **(b)** whether they will find it easy or hard when assigning a task* • *hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language*
	Knowledge of Content (Function Concept) and Teaching	• **(a)** know the different introductions for a particular topic, **(b)** know the different introductions for sequences of exercises, explanations, representations, definitions, methods, procedures and examples and **(c)** rate them in terms of their adequacy for particular learning groups • adapt their lesson planning in relation to changes in the class composition

3. Teachers' Knowledge of Function Concept

There are several studies in the existing literature dealing with teachers' knowledge necessary for mathematics teaching focusing on the function concept (Aksu & Kul, 2016; Even, 1993; Even & Tirosh, 1995; Hacıömeroğlu, 2006; Hatisaru & Erbaş, 2017; Karahasan, 2010; Nyikahadzoyi, 2015; Steele, Hillen & Smith, 2013; Wilson, 1992). For example, in her study, Even (1993) investigated teachers' subject-matter knowledge and its interrelations with PCK in the context of teaching the concept of function. Even and Tirosh (1995) also studied teachers' subject matter knowledge and students' knowledge of function concept, discriminating between "knowing that" and "knowing why". In addition, Wilson (1992) described PSMTs' knowledge and beliefs about functions and assessed the effects of some teacher education materials on the mathematical and pedagogical conceptions of teachers. Moreover, Hacıömeroğlu (2006) determined PSMTs' subject matter knowledge regarding the teaching the function concept and examined their PCK in order to reveal the relations between these knowledge domains. On the other hand, Karahasan (2010) examined PSMTs' of composite and inverse functions. Lastly, Aksu and Kul (2016) investigated mathematics teachers' knowledge of students about function concept.

It is also significant to point out that in recent years, researchers have been working on the concept of function with the MKT framework. Steele, Hillen & Smith (2013) focused on teachers' learning of prospective and practicing teachers in a teaching experiment consisting of a content-focused methods course involving the mathematical knowledge for teaching function. These researchers dealt with content knowledge domain within the MKT framework and connected it with other theoretical frameworks related to function concept. In another study, Hatisaru ve Erbaş (2017) have investigated the relationship between the mathematics teachers' MKT for the function concept and the learning outcomes of students for this concept. One of the studies dealing with MKT in the context of the teaching of the concept of function belongs to Nyikahadzoyi (2015). In Nyikahadzoyi's (2015) study, he unpacked the subject matter knowledge, teachers' PCK, teachers' technological pedagogical knowledge, technological content knowledge, and TPACK specializing in the concept of function. He proposed a framework by considering together MKT and the theories about function concept.

While these studies were examined in detail, we deduced that there are studies examining on in-service or pre-service teachers' knowledge specifically subject matter knowledge of the function concept as seen, but it could be said that the studies using the MKT framework are few and limited. The main goal of this study is to examine PSMTs' knowledge of the concept of function using and elaborating MKT framework while also extending the existing studies in this area.

4. Methodology

Qualitative research methodology was adopted in this study. In order to examine PSMT's MKT in terms of function concept, case study design which is described as 'examining a program, event, activity, process or one or more individuals in depth' was used in the study (Creswell, 2009). Particularly, the reason in choosing case study design is to deeply examine PSMTs' mathematical and pedagogical knowledge while they were teaching the function concept, so in order to analyze and improve their teaching as well as learning and to determine their view of teaching of function concept case study design was accepted as useful and efficient in this study (Zohrevand, Jafari, & Arshad, 2010).

4.1. Participants and Settings

This study was a part of a larger study. In order to examine and improve PSMTs' MKT for the basic calculus concepts (such as functions, the meaning of infinity, limit and continuity, sequences, series, derivative, integral and its applications etc.), a course named as "Teaching Calculus Concepts" was designed by considering the context of these concepts. Each week, PSMTs were responsible to design a lesson plan and to teach one of these concepts for 150 minutes. There were 27 PSMTs in the study, and they were grouped into 9 groups. The first week, three PSMTs were assigned to design a lesson plan and to teach the concept of fcuntion. This study focused on these three PSMTs' lesson plan and teaching process of function concept. In order to hide their identity, they were named as PSMT-A, PSMT-B, and PSMT-C. These PSMTs were female and they were at the fifth year (last year) in their program. Throughout their teaching, PSMT-A and PSMT-B took the heavy roles and PSMT-C only solved couple of examples at the end of the teaching process. Because of this, we mostly focused on PSMT-A and PSMT-B's responses, teaching styles and examples while giving the results of the study. Yet, since the main goal of the study was not to comparing PSMTs' teaching of function concept, these three PSMTs teaching was examined together. Throughout the teaching, since they designed the lesson plan together, they were able to change their roles, interfere their teaching, and teach together when they needed to.

Three PSMTs were responsible to lead the course for 150 minutes, and their classmates was asked to behave like a real high school student. In other words, the real classroom environment was created to evaluate PSMTs' MKT for function concept. Their classmates were also responsible for asking questions by considering what kind of questions high school students could ask and what kind of misconceptions regarding function concept might occur while learning of function concept. The PSMTs were able to use the smartboard, any material, technological tools or programs while they were teaching.

All the teaching process were recorded using a video camera. The camera was zoomed on the board when it was necessary to record PSMTs' writings, solutions, and responses more closely. In addition to this, two researchers sit at the different position of the class and took the observation notes while PSMTs were teaching. The researchers did not interfere PSMTs' teaching or did not ask any questions in order to not change the data.

4.2. Data Collection

In the study, the main data was gathered by observing and recording PSMTs' teaching of function concept as well as by examining their lesson plan of the concept. Video recording was used to examine structure of learning and teaching by watching videotape of PSMTs in action (Hall, 2000). Observation method was used to nurture and examine key pedagogical skills and PSMTs' teaching of the concept (O'Leary, 2014).

Before teaching the course, PSMTs were asked to design a lesson plan for function concept and share it with the researchers. The researchers did not interfere their lesson plans, it was only asked to be aware of teaching before observing it and to examine whether PSMTs followed their plan or changed the plan appropriately or inappropriately while teaching.

After the real classroom environment was created, PSMTs taught the function concept to their peers in the course that they took during the study. In this process, they were allowed to lead the class, ask any questions to their classmates regarding function concept, and change the lesson plan. In other words, they were asked to be the teacher of the course. During the course, the PSMTs taught one by one, but they sometimes interfered each other's teaching and they changed their roles. When one of the PSMTs could not answer the students' questions and could not give any response because of her excitement, the other PSMTs answered the questions or gave a response to the students. While the PSMTs were teaching, two researchers were observed the course and took some notes. Taking field or observation notes was important to record the moment that could not be identified by watching video records and to take instantaneous notes regarding their teaching. In summary, the lesson plan, video records of their teaching and observation notes constituted the data of this study.

4.3. Data Analysis

The data analysis process began by transcript the video-record of PSMTs' teaching, and this was the preliminary level of the analysis. The transcription document was 24 pages with single-spaced and they were desginated in order to easily follow the data in detail. The transcription was done since it plays a central role in research on spoken discourse distilling and freezing in time the complex events and aspects of interaction in categories of interest to the researcher (Edware & Lampert, 1993). The transcription document was analyzed using descriptive analysis. This method was used since it comprises of four stages which was used in this study: (1) constructing/combining framework/table, research questions, observation or interview notes, or theoretical framework(s) that were used in the study for descriptive analysis; (2) Analyzing the data using the framework; (3) definining the results; and (4) interpreting the results (Yıldırım & Şimşek, 2016). We used the MKT framework and its related frameworks and construct a table for analysis (See Table 1). Then, we analyzed the data by using this table, and defined and intepreted the results.

Two researchers initialy met and decided how to analyze the data using the constructed table. Then, they separately analyzed the first 10 pages of the transcription document and met again in order to check the percantage of agreement on the data. Since it was more than %70, they analyzed the remaining data together. For the first 10 pages, if they did not analyze the data using the same codes,

they tried to understand each other's perspectives and discussed on it. They revised the analysis until they had consensus on it. Then, they moved to the next level of analysis.
The lesson plan and observation notes were analyzed by using content analysis method. The main goal of using content analysis was to interpret the meaning of the content of the texts, tasks and responses that were given by the PSMTs (Creswell, 2013). Bryman (2004) described content analysis as 'an approach to documents that emphasizes the role of the investigator in the construction of the meaning of and in texts' (p. 542). In order to evaluate the meaning of the content of the lesson plan and tasks and questions that were used in the lesson plan, the content analysis method was used. Additionally, in order to emphasize our roles as observers in construction of the meaning of and in texts, we used the content analysis method. The observer notes were analyzed by using the transcription notes. The moments in the transcription notes and observation notes regarding these moments were synced and analyzed together.

The triangulation was used to ensure the validity of data. Specifically, we used different data sources such as lesson plan, observation notes and video-records. In order to ensure the reliability of the data, each researcher analyzed the part of the data separately. Then they met and discuss on it until we had a consensus on it. Additionally, after data analysis process was completed, the researcher met after a while and analyze the data again before reporting the results in order to ensure the relability of the analysis (Yıldırım & Şimşek, 2016).

5. Results

While PSMTs' teaching of function concept was examined, initially the transcript file was evaluated based on the subdomains of the theoretical framework. In this context, sufficient and insufficient situations for PSMTs' knowledge types were determined and moved into the tables. Then, their teaching process was analyzed in order to reveal the big part of the results in this study. Finally, their lesson plan was analyzed in order to examine their teaching of function concept.

5.1. General Perspective of PSMTs' MKT

It was determined 22 situations regarding PSMTs' CCK throughout their teaching of function concept. The 12 of these situations were determined sufficient while 10 of them were insufficient. While there was no evidence regarding subdomain CCK4, it was determined that the most stituations belonged to the subdomains CCK3 and CCK5, respectively. 62 different situations were determined regarding subdomain SCK throughout PSMTs' teaching of function concept. 23 of these situations were determined sufficient while 39 of them were insufficient. It was encountered that the most situations belonged to the subdomain SCK5. In regard to the subdomain KCS, 21 different situations were obtained in PSMTs' teaching of function concept. While 6 of these situations were sufficient, 15 of them were determined as insufficient. There were 34 situations for the subdomain KCT, and 12 of them were sufficient while 22 of the situations were determined as insufficient.

The genereal view of PSMTs' knowledge types based on the subdomains of MKT frameworks regarding function concept was shared in the above. In the following section, the results were presented including PSMTs' teaching process of function concept and the developed lesson plan, in detail.

Table 2. *General View of PSMTs' MKT*

	Sufficient	Insufficient
CCK1	-	1
CCK2	2	-
CCK3	9	-
CCK4	-	-
CCK5	1	9
CCK6	-	-
	12	10

		Sufficient	Insufficient
SCK1		2	6
SCK2	2a	4	1
	2b	4	2
SCK3	3a	3	3
	3b	-	-
	3c	1	-
SCK4		4	2
SCK5		1	21
SCK6		4	4
		23	39

		Sufficient	Insufficient
KCS1		-	7
KCS2	2a	3	2
KCS3	3a	2	-
	3b	-	-
KCS4		1	6
		6	15

		Sufficient	Insufficient
KCT1	1a	-	1
	1b	12	21
	1c	-	-
KCT2			
		12	22

5.2. Examination of PSMTs' MKT through Their Teaching Process

In this section, PSMTs' MKT was shared by considering the content of their teaching. Based on the content, the teaching was divided into two phases: (1) Constructing Function Concept; (2) Choosing examples. In these phases, each domain of MKT framework was examined in detail.

5.2.1. Constructing Function Concept

PSMTs began their teaching by using a readiness test in order to remind the prior knowledge that is neccessary for learning of function concept to students. Although, they indicated that they applied the test to the students in their lesson plan, they supposed that they completed the implementation of the test and directly began to teach the function concept. The PSMT-A began to teach and connected the function concept with the prior concepts such as sets, patterns, equation and inequalities, ordered pairs and Cartesian product. However, while she was doing that, instead of revealing the relationships between the function concept and these concepts, she only asked students whether they had deficiency knowledge of these concepts. This situation revealed that PSMT-A had difficulties in reflecting her knowledge (CCK2) of the relationships between function concept and other mathematical concepts to her teaching.

In the following step, PSMT-A presented a video that includes melting of an ice mountain and changes in a sandglass in order to start the teaching of the concept. By using this video, PSMT-A tried to constitute an intuition regarding concept of variation, which is also prior concept of function concept. Therefore, it was thought that she shared real life examples (CCK3) related to the function concept in her teaching. She also discussed these examples with the students and asked student to find different real-life examples in order to construct a meaning for the concept of variation in students' mind. Throughout her teaching, she shared another real-life example related to the function concept such as the numbers of workers and products, completed way depending on a time, and the relationship between the qantities of income and consumption. The class discussions were organized for the given examples. During the discussions, PSMT-A taught the definition of a function by using the terms including input-output, machine and the duty of the machine etc. After the dicsussion, she gave a rule and she paired two sets by using the rule. She also used different representation, venn diagram, to show the pairing. PSMT-A prequently reminded the real-life examples in order to show how and when a pair could be defined as a function.

PSMT-A reached the following definition while she was trying to teach the function concept: *A and B are two sets different from the empty set, a machine that matches an element of set A to an element of set B under some operations is called as a function*. This definition was spontaneously given to the students during the teaching even though it was not planned in their lesson plan. This definition presented an evidence for her knowledge regarding central definitions and properties of functions (CCK1). The definition that was given by PSMT-A was a correct definition, but it included a general expression. It was not a specific definition that was helpful to distinguish function concept from another mathematical concept. It was thought that this definition could be used in class discussions since a teacher, who aims to reach the definition of a function, could form students' thoughts step by step. Although some operations that was given in the definition was discussed in the class later, giving this definition on the board might cause limited and deceptive effects in students' minds. Therefore, it was deduced that writing the definition on the board by the teacher and writing it on notebooks by students showed that PSMT-A had struggle to reflect her knowledge (CCK1) regarding this sitution into her teaching. After the definition was given, the class discussion was done with the students in order to trigger them to find the term and some operations. PSMT-A examined the conditions that are neccessary to define a function by using the *input and output* and the worker-product example. While she was doing that, she mostly used the question-answer method. After examining the conditions, she obtained the following definition:

PSMT-A: *The function could be simply described as a machine that matches the inputs to the outputs. However, this machine is not an ordinary machine. This machine has some properties. We have to put all the elements in the input set into the operation in the machine, and what are others? An element in the input set cannot match with more than one element in the output set, and if and only if it matches with one element. Yet more than one element could match with the elements in the output set.*

Throughout the process of constructing function concept, some situations regarding PSMT-A's using of the language was taken as remarkable moments. For example, some of her explanations such as "Lets think about the time first time and second time" and "yes, our time is passing, let's say that 80 km is the output" had problems regarding using the language. The statement including the first time and second time was not understandable for the students. This situation revealed an evidence for her limited knowledge regarding using the terms and notations (CCK5). In addition to that, the statements such as "one variations could change depending on one another variation" or "This matching, the arrow that matches this could be named" show that she did not use the mathematical language correctly (SCK5).

While constructing function concept, PSMTs shared evidence related to their knowlede of representations and using them. For example, while trying to construct function concept, PSMT-A firstly showed the way, which a car completed, in the table with a student. Then, she moved the table into the Venn diagram and preferred to use different representation. Thereby, she was able to choose different representations as well as move move flexibly between two representations regarding function concept. This situation showed that PSMT-A was able to reflect her knowledge of specialized content knowledge (SCK2) into her teaching. In the other situation, she pointed out that a matching, which was presented with a Venn diagram, could be stated as a machine:

PSMT-A: *At the same time, if I call this set A as inputs, in short, I changed the name of it as inputs. I also changed the name of set B as outputs. Here, the name of the arrow that matches the sets has a name; it was a 'star' or just 'a'. It does not matter. I think about that as a machine. In other words, there is a machine that takes the time and matches it with the taken way. I named the operation between sets as the machine. Now, what is the duty of our machine in here?*

Although PSMT-A tried to switch to different representations, she was not able to construct the relationships between two representations. After PSMT-A reached the general definition for function concept, she examined the rules that are necesseray to define a function. After she reached the definition, she also preferred to show the function as algebraically. However, she had difficulties in this point. Moreover, during her teaching, she discussed it with her group mates; she silently asked, "I will pass the function f(x), but how I will do it?" After that, PSMT-B continued to the teaching and she showed it by using the algebraic representaion of the function. This situation revealed that PSMT-A had a limited knowledge in regards to specialized content knowledge (SCK2-b).

Throughout the process of constructing the concept, PSMT-A took an active role and she mostly adopted the approach regarding inputs and outputs and the explanation and examples regarding the machine. It other words, PSMT-A conceived function concept as a procedure or a process. However, after she reached the definition of a function concept, she had struggle to describe the concept mathematically which shows that she had a limited knowledge regarding transforming her knowledge in to the object level.

It was indicated while PSMT-A tried to construct the function concept, she created environment in order to do class discussions and she asked some questions to the students. It was thought that it was significant to examine how she managed this process and how she figured out the students' thinking of function concept, in detail. In general, when PSMT-A asked questions to students, she preferred to take couple of answers from the students and continued on using these answers. She did not try to understand students' thoughts in depth, and she did not give an opportunity to students for constructing the concept. In many situations, she did not interpreted the students' answers. For example, in the following dialog, PSMT-A asked students to intrepret the similarities and differences between two videos that they watched. For this purpose, PSMT-A asked questions to students, and requested to answer these questions. However, she did not interpreted students' answers. This situation provides evidence for her knowledge of KCS4; she was not able to hear and interpret students' emerging and incomplete thinking as expressed in the ways that students used language.

PSMT-B: *Does anyone would like to explain the relationship between them?*
Student: *The time and sandglass.*
PSMT-A: *Yes. What happened in the other one?*
Student: *Ice mountain melted with heat.*
PSMT-A: *If we woul like to construct a connnection, how can we do that?*
Student: *Interconnected.*
PSMT-A: *How does it interconnect to each other, could you describe it?*
Student: *In short, there is a factor, and changes are done based on this factor.*
PSMT-A: *Yes, it is true. Does anyone woul like to share something?*

In addition, PSMT-A did not give enough explanation throughout her teaching. The answers that was given based on students' questions or her interpretations regarding those questions showed her limited knowledge regarding teaching (specifically KCT1b). It is thought that these situations might depend on many factors such as teaching was not conducted in a real classroom environment, the students were their classmates, so they could not act on like a real student, and PSMTs' had limited (almost non) experiences during the teaching.

5.2.2. Choosing Examples

Examples play a central role in learning of mathematics (Watson & Mason, 2002). Therefore, choosing good/neat/suitable examples play a key role in mathematics education. In this study, PSMTs began their teaching of the function concept by giving real-life examples. They focused on critical aspects of the concept specifically on the examples of the correspondence between the workers and the products and the way that was taken depending on time. While examining sub concepts of the concept of function, PSMTs derived from the example of "workers-products" during their teaching. But this example created a problem for students' understanding of the concept throughout the teaching. For instance, PSMTs sometimes corresponded the number of the workers to the number of the products, sometimes the workers' names to the number of the products. Therefore, we deduced that this example was not a good example while teaching the definition of the concept. It could be beneficial to consider the problems that it might cause while designing the lesson plan.

After PSMTs adopted input-output approach by giving real-life examples at the beginning of the lesson, PSMT-A gave a mathematical example for the concept of function as follows:

PSMT-A: *Let our machine has a function and this function be x^2. The variables namely inputs be 1, 2, 3. And think that our outputs are 1, 4, 9.*

This was the first mathematical example after the real-life examples was given, but PSMTs did not reach the definition of the concept of function by using the example. The example was used at this moment could be an easier one instead of $f(x)= x^2$. In choosing examples, following a path from easy to hard and basic to complex could be a suitable way for promoting students' conceptual learning of functiıb concept. Therefore, PSMT-A's using/choosing this example indicated insufficiency knowledge regarding the content (function concept) and the teaching (KCT1b). In spite of this insuffiency, she could emphasize the importance of the mathematical rule for whole concept (SCK3b) and could explain it to the students at the same time (SCK3c). PSMT-A' explanations had also shown that she followed a positive instructional path (KCT1b) as it is given:

PSMT-A: *Because our machine can be anything. What did we say when we used this machine as an example of real-life? We mentioned one thing, for example, about the textile workshop. What did we say at the textile workshop? The work done by the workers was the outcome. For example, what was our machine doing? Products. We had one machine that we made. So, it does not need to be a number like x^2. It does not have to be related to the numbers. It is also the duty of this machine to perform those jobs in the real-life, such as those products. So, you can change that machine's mission.*

As stated before, PSMT-A had difficulty in switching mathematical representation of the function concept and PSMT-B interfered to the teaching. She began her words with using an analogy for the function concept, "a truck carrying products". But the word of truck had been a wrong choice for representing the function concept. Hence, a student said that a product will stay the same whether it was carried by a truck or not. Then, PSMT-A corrected and said that the Word of "packing machine" would be more appropriate rather than truck. Following statement of PSMT-B is an evidence for this finding.

PSMT-B: *Let's think of it as a factory A and a factory B. In the meantime, we have a truck. Let's say you're going to be put in a different package there. What is the duty of the truck here? Take this element (showing set A) and take it here (showing set B). When we look at our A and B sets like this, we have our domain and range sets, f function, that is, function f. Here is the processor symbolizing the trucks. What is it doing? It takes the elements in A and matches them to set B. If we express the elements in A as x, let's express these as y. For example, x_1, x_2, x_3. These are y_1, y_2, y_3. What happens then, Hatice? f takes x's to y. And I can symbolically express this in the following way. If x's are generally expressed as x, y's are expressed as y, then f (x) is equal to y. Is this always supposed to be like this? No. Here (showing the domain set) y_1, y_2, y_3 are also here (show the range set) x_1, x_2, x_3. What happens then, Muhammed? f takes y to x. f(y) is equal to x (responses to class are very fast). We had a general round-up okay?*

At the above quotation, PSMT-B asked some questions to the students (Hatice and Muhammed) but she did not wait for the answers. She gave the answers by herself quickly. Indeed, she gave the algebraic form of the function concept as x=f(y) rather than y=f(x). This could cause some misconception for students and the students might find this representation confusing. This approach of the PSMT-B could be commented as an insufficiency regarding to the knowledge of content and students (KCS1). After the algebraic representation of the function concept, PSMT-A attempted to represent it via list method (See Figure 2).

PSMT-A: *If we show it as the last list. For example, we said f. Let's say general rather than x. For example, we took this set as A. Where are they taking the elements of this set then? Then, take a minute (look at the notes in your hand). I take my f(x) (write on the board). So, what I call f(x)'s f (A), ie f (A), is the function that takes a set of A from a set of B, my f(x) that I took x's gave whole of the set. We can do this with the list method as well.*

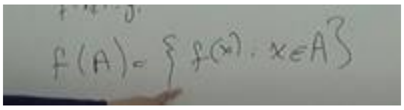

Figure 2. *The figure that was given to show the function by using list method*

However, while trying to do this, it was striking that she was uneasy and anxious. This finding revealed evidence for that PSMT-A was very unexperienced in teaching. Then, she looked at her notes and tried to show list method for the function concept. But she could not success on showing it. Therefore, the statement that was given to show the function concept by using list method was not a representation of function concept. It was the image set written by common feature method. She not only said the name of the representation wrongly but also could not explain the mathematical statement on the board. PSMT-A could not use her knowledge about definitions of the function concept properly (SCK1) and not use mathematical language in a suitable way (SCK5).

In teaching, PSMT-B gave examples about the concept of function as Figure 3. During this process, there was an evidence that she was insufficient in using mathematical notations (CCK5). In the quotation below, had a mistake on representing a set and corrected her mistake by a student's intervention.

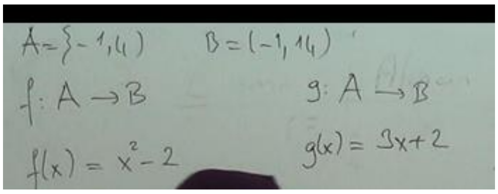

Figure 3. *PSMT-B' example*

Student: *Now we had three rules. First ... So, f is from A to B and g is from A to B, too. That is domain and range sets; they are the same.*
PSMT-B: *Yes.*
Student: *I need to look at 3. Every x element is for A ...*
Student: *Is that something a closed interval?*
PSMT-B: *It is not a closed interval, it is a representation of the set. A lot changes.*

PSMT-C continued teaching by giving some examples tegarding function concept. Initially, PSMT-C reminded the conditions that are neccesarry for definining a function and showed some examples of correspondences via Venn Diagram. But while doing this, she drew one Venn Diagram on the board and showed three different correspondences on the same diagram by erasing the previous one and writing the new (probably in order to use time efficiently). Whereas, if she preferred to draw three Venn Diagrams separately on the board, she would be able to examine the common and the different points of these correspondences. This finding could be seen as a reflection of PSMT-C's insufficiency of KCT.

On the other hand, when PSMT-C wrote the below example on the board (See Figure 4), a discussion began in the class.

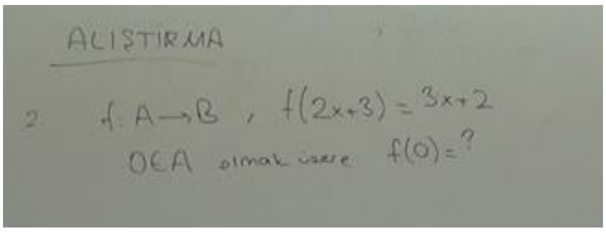

Figure 4. *PSMT-C's example*

PSMT-C: *We are looking for f(0). This is the function that takes to 3x+2. Let me show (on the board). I'm trying to figure out where 0 goes. So, what do I have to do then? Can we get out of it if I try to equalize them? The value that makes 2x+3 zero is -3/2.*

Student: *Teacher, how do I know that -3/2 is in set of A or not? 0 is a member of the A but does not it (-3/2) have to be A?*

As seen in the quotation, the rule that the students were familiar to be given as f(x), was f(2x+3)=3x+2 at this time. This example might be different and complicated for students and they were confused about which element the domain set was made of. PSMT-B who could conceived the function concept as procept (SCK4), interfered the teaching and tried to explain the example to the students by the below words:

PSMT-B: *Something like this, normally the domain set have x's. Now the form of elements in our domain set has changed. The elements in our domain set are numbers like 2x+3. We have found x as -3/2 here, but we do not see if it is in this domain set or not. It does not have to be if we take a look. The numbers in the 2x + 3 format are in domain set. Not the x's.*

PSMT-B's explanation in the above quotation was still not enough for students to be convinced, PSMT-C did an explanation again. But this one had also been insufficient (KCT1b). It was given in the below, and we inferred that PSMT-C's usage of mathematical language explicitly was also insufficient (SCK5).

PSMT-C: *Ok, I will repeat, function is a function that transforms variables of 2x+3 into variables of 3x+2. I want to find the image of 0 here, that is, what the 0 changes under f. I know that in finding it we have to make 2x + 3, zero. Since the value that makes it 0 is also the value of -3/2, I am looking for 3x+2 here too. What will happen to 3x+2 if x is 3/2? f(0) … (solves for itself, without asking questions to students, towards the board).*

In the section presented above, there was a new function (f(2x+3) =3x+2) obtained by transformation of the old one. This mathematical example was complex for students to understand and hard for PSMTs to explain. Already, it caused a discussion in the classroom. Therefore, it was thought that it could be more appropriate to begin with an easier one and go ahead by reasoning on it while dealing with transformed functions. This example selection of PSMTs was considered to be an insufficient condition for KCT.

PSMTs completed their teaching the function concept by playing a game in class. The game was played in groups, and they divided the class into four groups. Each group initiallt identified a function. Groups were asked to find other groups' functions. Therefore, each group was asked to write the ordered pairs that provide this function to the board. During the the game, there was usually chaos in the class, the groups spoke loudly among themselves. PSMTs were not sufficiently managed the group discussion and class environment during the game. When there are two different functions that provide an ordered pair during the game, the PSMTs could not predict that such a situation might occur, and the game is terminated. It is thought that such a game should be played to support the construction of the concept of function by taking advantage of ordered pairs and rule concepts in the first stage of function concept teaching.

5.3. Lesson Plan

The PSMTs planned two lesson hours in order to teach function concept, specifically the meaning and definition of the concept. In their plan, the purpose was determined as "Explaining the concept of function.". They stated that the subject is related to other disciplines (physics, chemistry, biology, geography). Some real-life examples that they gave in their teaching can be said to fall into the fields of other disciplines, but they could not elaborate this connection in detail.

PSMTs' lesson plan included couple of objectives in a comprehensive manner. However, PSMTs were not able to reflect all the objectives into their teaching. In addition, they mentioned about the learning-teaching method techniques, the tools and equipment to be used and the learning-teaching activities in detail in their plan but they did not adequately reflect this to their teaching. In addition to that, although there were comprehensive types of readiness test, games, and different types of questions in the lesson plan, some problems occurred while applying them. For example, although the game in Figure 5 was given in the lesson plan, they did not play this game in their teaching. This suggested that there were some problems in transferring the theoretical knowledge of PSMTs and the lesson plan they designed into practice.

- We will ask the students to write down the results of some of the numbers they hold to the students by saying "Keep a number from the mind". And we want them to relate to the function by showing it with a schematic.

Figure 5. *A screenshot from PSMTs' lesson plan*

6. Discussion and Conclusion

In this study, our main goal was to examine PSMTs' knowledge and their teaching of the concept of function within the MKT framework. For this purpose, we analyzed their teaching process considering their knowledge types and lesson plan. The results of the study showed that PSMTs' had limited knowledge of function concept concept based on MKT framework. Consistent with the existing studies (Even, 1993; Karahasan, 2010), the findings of this study revealed that PSMTs' knowledge domains regarding MKT framework were not as the expected levels.

It is thought that PSMTs' limited knowledge effected their teaching of function concept similar to Hatisaru and Erbaş's (2017) study. Particularly, while constructing function concept, PSMTs used real life examples, and they tried to form the function concept into these real-life examples. However, while giving the definition, they preferred to begin with a general definition which could cause confusion on students' minds. Then, they discussed the conditions that are necessary for defining a function by using given real life examples. Finally, they reached the definition for function concept, but they used incorrect terms, notations or mathematical language while defining it.

When PSMTs' choices of examples were examined, it was deduced that they mainly used real-life examples during their teaching. However, some of these examples did not consider well and they did not explain or discuss them, in advance. In addition to the mathematical examples, they generally used complex or confusing examples. Particulary, they did dont follow a way from easy to hard in their examples.

During the teaching process, even though PSMTs mostly asked questions to student, they did not give an opportunity to them to response these questions or they did not confirm or interpret the answers. Similar to Aksu and Kul's (2016) study, mathematics teachers had limited knowledge on responding, hearing or interpreting students' answers. Moreover, PSMTs had some difficulties in making explanation in regard to students' questions throughout the teaching process. These results illuminated PSMTs' limited knowledge of MKT especially regarding SCK1, SCK5, CCK5, KCS1, KCS4, and KCT1.

The ability to design a lesson plan is required to using teachers' teaching methods, using tools that are used in the teaching process, having a knowledge to manage procedural and conceptual knowledge in activities, having the ability to use these tools and activities in right places, and constructing a model that is appropriate for their goals (Kolb, 1984). As it was mentioned in the Kablan's (2012) study, the process of designing a lesson plan is an important process for teachers. In this study, the lesson plan designed by PSMTs was comprehensive, so it was thought that they achieved an important step based on these researchers' suggestions regarding designing a lesson plan. Therefore, it is deduced that it is important to reflect the plans to their teaching. We thought that it is significant to design PSMTs' education programs by considering development of their teaching. In order to examine the effect of education programs in their teaching, one could design a course (within the education program) to improve their teaching, and look for the impact of the program in their teaching of function concept.

When all the results were considered together, it was thought that PSMTs' struggles in teaching of function concept is directly related to their unexperience of teaching. It is possible to manage time and classroom environment and communicate with students much better when PSMTs' do more practice and become experienced. In other words, it is thought that one of the factors that affect PSMTs' limited knowldge is experience (Wongsopawiro, 2012; Van Driel ve Berry, 2012). On that account, it is important to gain experience to them. For example, they could be given more chance to practice or teach in the courses by creating a real class environment, and more collaborations could be done with real schools, so they could have a chance to watch these classrooms and teach in there. Future research could be done in these real classes to determine PSMTs' knowledge within the MKT framework.

In conclusion, if PSMTs are supported appropriatelly and are prepared for effective teaching, they could improve their knowledge of teaching function concept as it was indicated in the existing studies (Morris, Hiebert & Spitzer, 2009; Steele, Hillen & Smith, 2013; Wilson, 1992). Therefore, it is important to support PSMTs either to improve their theoretical knowledge or to encourage them to reflect this knowledge on their teaching in order to develop their MKT. Future research could be designed by considering the importance of experience and an ability to reflect the knowledge on the teaching in order to improve both in-service and pre-service teachers' MKT.

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