



EXAMINATION OF MATHEMATICS TEACHER CANDIDATES' STRATEGIES USED IN SOLVING NON-ROUTINE PROBLEMS

Melihan Ünlü

Abstract: The aim of the study was to examine mathematics teacher candidates' strategies used in solving non-routine problems. The research was carried out with 104 mathematics teacher candidates studying at a state university during the first term of the 2014-2015 academic year in Turkey. Data was collected through Problem Solving Test which was consisted of 10 non-routine problems. Descriptive statistics were used to determine mathematics teacher candidates' solutions and strategies used in problem solving. Solutions of problems were rated and categorized. Firstly, solutions were classified as correct or wrong. Secondly, correct and wrong solutions classified as "solving by using true strategy" and "solving by using wrong strategy". After that "solving using true strategy" was categorized as "indicated strategy correctly" and "indicated strategy wrongly". The findings of this research revealed that some of mathematics teacher candidates have adequate problem solving skills but some of teacher candidates did not solve problems correctly because they used wrong strategy. In other words, mathematics teacher candidates have limited abilities at the stage of "understanding of problem" and "planning". It was a due of they could not understand what the problem says and they could not choose an appropriate strategy for problem solving. Some of teacher candidates used correct strategies in solving problems, even though they could not reach the correct answer. It was an evidence that these teacher candidates have lack of knowledge about "applying plan". On the other hand, some of teacher candidates could solve problems correctly but they could not indicate strategy correctly.

Key words: problem solving strategies, non-routine problems, teacher candidates.

1. Introduction

Problem solving is a learning approach that can be used to improve the quality of teaching and remove many learning difficulties (Altun & Arslan, 2006). Therefore, many researchers focus on problem solving over the last decades. Problem can be defined as a question or a situation that creates confusion and uncertainty in the individuals' mind (Posamantier & Krulik, 1998; Sheffield & Cruikshank, 2005). In other words, problem is a situation that has an unusual solution, few different knowledge and skills are used together to solve it (Ministry of National Education [MoNE], 2006). For the conceptual development of students, the type of problem is important (Ross & Kennedy, 1990). Van De Walle, Karp and Williams (2012) stated that problems which are used in mathematics learning should have certain characteristics. Accordingly, a problem must be compatible with students' prior knowledge, compelling and interesting. Moreover, the interesting aspect of the problem should be also based on the mathematics and the problem should be able to explain the correctness of the answers and why it is correct. In National Council of Teachers of Mathematics (NCTM) Standards (2000), it is stated that good problems are "arising from students' environment", "forcing the students to develop their own strategies and practices" and "preparing the environment for introducing new concepts to students". Problems are classified as routine and non-routine problems. While routine problems can be solved easily by applying four operations and certain rules, non-routine problems require higher level thinking (Arslan & Altun, 2007; Inoue, 2005) and improve students' tendency to examine events and search for relationships, order or pattern (Altun, 2008). Since non-routine problems are non-standard, involving unexpected and unfamiliar solutions, students are generally afraid of the idea of solving non-routine problems (Apostol, 2017). On the other hand, Olkun, Şahin, Akkurt, Dikkartın and Gülbağcı (2009) classified problems as standard verbal problems and non-standard verbal problems. Standard verbal problems are problems that can be solved by applying one or more

Received May 2018.

Cite as: Ünlü M. (2018). Examination of mathematics teacher candidates' strategies used in solving non-routine problems. *Acta Didactica Napocensia*, 11(3-4), 97-114, DOI: 10.24193/adn.11.3-4.8.

arithmetic operations, whereas nonstandard verbal problems require special considerations as well as the application of arithmetic operations.

Problem solving is defined as the way to reach the solution in situations where solution is not known (Polya, 1962). It is not used in low level learnings and situations where learners know what to do (Schunk, 2009). Problem solving is a process and in this process, students should be presented with environments where they are creative, use different strategies and create new problems rather than learning and implementing the algorithm and rules (MoNE, 2006; Van De Walle, Karp & Williams, 2012). Furthermore, students need to develop strategies for problem solving in order to learn how to solve problems (Baykul, 2009, MoNE, 2006). For that reasons, it is important for students to face non-routine problem situations while developing their problem solving skills (Olkun et al., 2009). Because students think flexible and practical when solving problems which they have not encountered before (PISA, 2005). Therefore, starting with simple problems and then providing students an environment where they can learn problem solving strategies with more difficult and complex problems will also improve students' problem solving skills (Posamantier & Krulik, 1998).

In problem solving process, Polya (1990) focused on four problem solving steps: understanding the problem, planning, applying the plan and evaluating the solution. Understanding the problem is the first step of the problem-solving process. In this step, it is crucial to understand which information is given, what is happening in the problem and what is required for solution. The second step is making a plan. In this phase, a solution plan is made considering how to solve the problem, a strategy or strategies for solution is selected. Problem solver should choose an appropriate strategy because using appropriate problem solving strategy is important in terms of being successful in problem solving (Ersoy & Güner, 2014). Posamantier and Krulik (1998) classified problem solving strategies as working backwards, finding a pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing (visual representation), intelligent guessing and testing, accounting for all possibilities, organizing data and logical reasoning. If there is unique endpoint and variety of paths to get the starting point by working back, working backwards strategy is suitable for solution. Finding a pattern is a strategy that problem solver seeks a pattern and uses this pattern to solve problem. In adopting a different point of view strategy, it is required to look at the problem from different perspective. Solving a simpler analogous problem strategy is a strategy that problem solver changes the given problem into one that may be easier to solve. Considering extreme cases strategy is used solving problem such as where some variables are constant and others are varying to extremes. In making a drawing strategy (visual representation), it is used diagrams or drawings to see relationships between situations and problem solver can solve problems according to these drawings. Intelligent guessing and testing is a strategy that problem solver guess the solution and test to show it is correct or not. In organizing data strategy, given data from the problem situation is reorganized in a way different from the way it was presented. Accounting for all possibilities is a strategy that problem solver considers all options and chooses the most suitable one. In logical reasoning strategy, logical reasoning is a thinking process. For an example, if you say A then it is expected that the response will B. This strategy also helps to make proof (Posamantier & Krulik, 1998). The third step of problem solving process is applying the plan. In this stage, the prepared plan is applied and solution is made by using the determined strategy. In the evaluation phase, the solution is checked whether the answer is really correct or not (Polya, 1990). The lack of one of the problem-solving steps will result solving the problem wrongly.

According to the researches, many students tended to apply the essential procedures to given numbers and find solution instead of cognitive activities such as judging the solution process, analyzing the problem, evaluating the results (Arslan & Altun, 2007). Aksoy, Bayazit and Kırnay-Dönmez (2015) also stated that majority of the teacher candidates tended to use rules and procedures in a straightforward way and lacked the ability to use appropriate strategies that could scaffold their realistic considerations. Yeo (2009) examined secondary 2 (13-14 years old) students' difficulties in solving non-routine problems. According to the results, the difficulties experienced by students were lack of comprehension of the problem posed, lack of strategy knowledge, inability to translate the problem into mathematical form, and inability to use the correct mathematics. Ulu (2011) also stated that during the solving non-routine problems, primary school students made mistakes about reading

and comprehension (49.26%), strategy-based errors (8.08%), (from strategy selection and execution of strategy) and operation errors (2.62%).

The increasing of emphasis on problem solving in mathematics education has necessitated the researches on problem solving processes (Gür & Hangül, 2015). Many researches focused on problem solving (Aksoy, Bayazit & Kırnay-Dönmez, 2015; Che, Wiegert, & Threlkeld, 2012; Çeker & Ev-Çimen, 2017; Çelebioğlu & Yazgan, 2009; Elia, Heuvel-Panhuizen & Kolovou, 2009; Gökkurt-Özdemir, Erdem, Örnek & Soylu, 2018; Gökkurt-Özdemir, Koçak & Soylu, 2018; Gür & Hangül, 2015; Olkun et al., 2009; Türker-Biber, Aylar, Sonay-Ay & Akkuş-İspir, 2017; Verschaffel, De Corte & Lasure, 1994; Yazgan & Bintaş, 2005). Researches on problem solving strategies investigated the strategies generally used by teachers, teacher candidates and students in problem solving process. Yazgan and Bintaş (2005) found that fourth and fifth grade students mostly use intelligent guessing and testing strategies; least finding pattern, working backwards and making drawings. However, Altun and Arslan (2006) indicated that 7th and 8th grade students were most successful in using systematic listing, making drawings, and working backwards strategies. When the literature is examined, there were limited researches which directly questioned to examine strategies used by mathematics teacher candidates in solving non-routine problems. Because teachers can only be trained as good problem solvers by learning problem solving strategies and how to use them (Posamantier & Krulik, 1998). In addition, teacher candidates' knowledge about problem solving and problem solving strategies will be efficient while teaching these topics effectively to their students. Although problem solving and problem solving strategies are concepts that students have encountered at many fields and class levels, the importance of these issues and the difficulties that students have experienced in the problem solving process, make it necessary to conduct research on this subject. On the other hand, developing problem-solving activities and teaching strategies to their students are up to teachers who have knowledge and experience in this area. Therefore, it is important that teacher candidates should be equipped to perform problem solving activities for their students. In this context, it is thought that examining the problem-solving processes of the mathematics teacher candidates who will be responsible for teaching this topic to the students in the future, is important for mathematics education. This research will also determine whether mathematics teacher candidates were aware of the strategies they used in problem solving or not. For this purpose, the aim of the study was to examine mathematics teacher candidates' strategies used in solving non-routine problems.

2. Method

2.1. Research model

Descriptive survey method was used in this research. In survey methods, information is collected from a group of people to describe some aspects or characteristics (such as abilities, opinions, attitudes, beliefs, and/or knowledge) of the population (Frankel & Wallen, 2005). Since the aim was to examine mathematics teacher candidates' strategies in solving non-routine problems, survey method was chosen.

2.2. Participants

The research was carried out with 104 mathematics teacher candidates studying at a state university during the first term of the 2014-2015 academic year in Turkey. 28 teacher candidates were in their second, 37 were in their third and 39 were in their fourth grade. There were 82 female and 22 male students enrolled the study. Purposeful sampling method was used to select participants. Teacher candidates had enrolled Mathematics Teaching I method course in third year and some had enrolled elective Problem Solving Strategy course in their second year. Within the scope of Mathematics Teaching I and Problem Solving Strategy; problem, problem solving, Polya's problem solving stages, routine and non-routine problems, problem solving strategies and their applications were covered. Teacher candidates were gathered essential information about problem solving strategies.

2.3. Data Collection Tools

Data were collected through Problem Solving Test which was consisted of 10 open ended non-routine problems. Problems were adopted to Turkish from “Problem solving strategies for efficient and elegant solutions” book (Posamantier & Krulik, 1998). Each problem is associated with a different strategy. First problem is about working backwards strategy, second is about finding a pattern strategy, third is about adopting a different point of view strategy, fourth is about solving a simpler analogous problem strategy, fifth is about considering extreme cases strategy, sixth is about making a drawing (visual representation) strategy, seventh is about intelligent guessing and testing strategy, eighth is about accounting for all possibilities strategy, ninth is about organizing data strategy and tenth is about logical reasoning strategy. In Problem Solving Test, mathematics teacher candidates were asked to solve problems using appropriate problem solving strategies and to indicate the strategies that they used in problem solving process.

Firstly, 10 problems were translated into Turkish by the researcher and three field experts. Then test was applied to the 10 teacher candidates to check problems whether they were understandable or not. After the necessary arrangements from the feedbacks, the Problem Solving Test was finalized.

2.4. Data collection and analysis

Problem Solving Test was applied to teacher candidates. In the Problem Solving Test, it was required to solve the given problems by using appropriate strategies and to indicate the strategies that they used for their solutions. There were not any time constraints during the test and necessary precautions were taken to ensure that participants were not affected by each other.

Descriptive statistics were used to determine mathematics teacher candidates’ solutions and strategies which used in problem solving process. Solutions of problems were analyzed by two researchers independently from each other. Then ratings were compared. When there was discrepancy between classification, researchers discussed and reached to a consensus. Firstly, solutions were classified correct, wrong or blank.

If teacher candidates understood the problem correctly, planned the solution, chose correct strategy and solved problem correctly by using correct strategy, the solution was evaluated in the “correct solution” category. Then, correct solutions were classified as “solving by using correct strategy” and “solving by using wrong strategy”. Afterwards, correct solutions were classified as “indicated strategy correctly” and “indicated strategy wrongly” according to whether or not the teacher candidates correctly state which strategies they used in problem solving. For example, solution for first problem: *“Ali loses in round 3 and gives Esen and Huriye as much money as they each have. At the end of the game Esen has 24 TL and Huriye has 24 TL. This means before round 3 Esen and Huriye has 12 TL, and Ali has 48 TL. Huriye lost in the second round. This means before round 2 Huriye has 42 TL (She gave 6TL to Esen and 24 TL to Ali). In round 1 Esen lost so she gave 21 TL to Huriye and 12 TL to Ali. From that Huriye has 21 TL, Esen has 39 TL and Ali has 12 TL at first.”* In this solution, teacher candidate solved problem by using working backwards strategy. She reached the correct answer and indicated that *“we can solve the problem with working backwards strategy”* so it was classified as “correct solution-using correct strategy-indicated strategy correctly”. In another solution *“At the end of game Esen, Huriye and Ali has 24 TL; in round 2 Esen has 12 TL, Huriye has 12 TL and Ali has 48 TL; in round 1 Esen has 6 TL, Huriye has 42 TL and Ali has 24 TL. At first Huriye has 21 TL, Esen has 39 TL and Ali has 12 TL.”* In this solution, teacher candidate used working backwards strategy but he did not aware of strategy which he used. He indicated that *“I used considering extreme cases strategy”* so it was classified as “correct solution-using correct strategy-indicated strategy wrongly”. If teacher candidates understood the problem correctly, planned the solution, chose correct strategy and solved problem correctly by using correct strategy but he/she did not indicate problem solving strategy what she/he used, the solution was evaluated in the “correct solution-using correct strategy- not indicated strategy” category.

If teacher candidates chose correct strategy but solved problem wrongly, the solution was evaluated in the “wrong solution” category. Then, the wrong solutions were classified as “solving by using correct strategy” and “solving by using wrong strategy”. For example, solution of first problem: *“We can use*

working backwards strategy. Ali loses in round 3 and gives Esen and Huriye as much money as they each have. At the end of the game Esen has 24 TL and Huriye has 24 TL. This means before round 3 Ali has 48 TL, Esen and Huriye has 12 TL. Before Huriye gave money to Esen and Ali, Esen has 6 TL, Ali has 24 TL and Huriye has 42 TL. If we continue like that, Huriye has 21 TL, Esen has 75 TL and Ali has 48 TL at first." In this solution, teacher candidate solved problem by using working backwards strategy but she did not reach the correct answer so it was classified as "wrong solution-using correct strategy". If teacher candidates chose wrong strategy and did not solve problem correctly it was classified as "wrong solution-using wrong strategy"

In order to verify the reliability of the Problem Solving Test, teacher candidates' problem solving sheets were analyzed separately by a second encoder (an academic member from the educational field of mathematics). The accordance between the researcher and the second encoder was calculated at .92, by using the Consensus/(Consensus + Dissensus) X 100 formula offered by Miles and Huberman (1994). As a result of the analysis, categories were converted into frequency and percentage tables.

3. Findings

Teacher candidates' solutions were classified as correct solution, wrong solution or blank. In the next phase, solutions were categorized whether they used correct strategy or not. After that "solving by using correct strategy" were categorized "indicated strategy correctly" and "indicated strategy wrongly". Afterwards wrong solutions were categorized based on whether they were indicated strategy correct or wrong. Frequencies (f) and percent (%) of solutions were given in Table 1.

Table 1. Examining teacher candidates' problem solving process

Problem number	Problem strategy	Correct Solution						Wrong Solution				Blank	
		Using by Correct Strategy						Using by Correct Strategy		Using by Incorrect Strategy			
		Indicated Strategy Correctly		Indicated Strategy Wrongly		Not Indicated Strategy							
		f	%	f	%	f	%	f	%	f	%	f	%
1.	Working Backwards	34	32.7	5	4.8	-	-	16	15.4	41	39.4	8	7.7
2.	Finding Pattern	57	54.9	1	1	4	3.8	19	18.3	12	11.5	11	10.5
3.	Adopting a Different Point of View	37	35.6	5	4.8	2	1.9	2	1.9	36	34.6	22	21.2
4.	Solving a Simpler Analogous Problem	29	27.9	3	2.9	44	42.3	1	1	16	15.4	11	10.5
5.	Considering Extreme Cases	85	81.7	5	4.8	-	-	8	7.7	4	.8	2	1.9
6.	Making a Drawing	67	64.4	18	17.3	-	-	6	5.8	8	7.7	5	4.8
7.	Intelligent Guessing and Testing	89	85.6	5	4.8	-	-	3	2.9	5	4.8	2	1.9
8.	Accounting for All Possibilities	44	42.3	4	3.8	-	-	1	1	45	43.3	10	9.6
9.	Organizing Data	52	50	1	1	-	-	19	18.3	17	16.3	15	14.4
10.	Logical Reasoning	4	3.8	-	-	1	1	1	1	85	81.7	13	12.5

According to the Table 1, teacher candidates were most successful in intelligent guessing and testing, considering extreme cases and making drawing strategy; whereas they were the less successful in logical reasoning strategy. They were also successful in solving a simpler analogous problem, finding a pattern, organizing data, accounting for all possibilities, adopting a different point of view and working backwards strategy respectively.

3.1. Results about Working Backwards Strategy

First problem is “Esen, Huriye and Ali play a certain game. The player who loses each round must give each of the other players as much money as the player has at that time. In round 1, Esen loses and gives Huriye and Ali as much money as that each have. In round 2 Huriye loses and gives Esen and Ali as much money as they each then have. Ali loses in round 3 and gives Esen and Huriye as much money as they each have. They decide to quite at this point and discover that they each have 24 TL. How much money did they each start with?” (Working Backwards). This problem is about working backwards strategy. Also, this problem can be solved with solving equation, but this strategy makes the solution long, difficult and complex. According to the Table 1, 37.5% of the teacher candidates solved the problem correctly. 32.7% of them indicated strategy correctly, 4.8% were wrongly. 54.8% of teacher candidates solved problem wrongly. 15.4% did not find the correct answer while using correct strategy. It was an evidence that these teacher candidates have lack of knowledge about “applying plan”. 39.4% of them did not reach the correct answer since they used the wrong strategy and 7.7% did not make any solution to the problem. Teacher candidates generally used the strategy of working backwards and solving equation in problem solving. Examples of solutions were given below:

Bu soruyu geriye doğru stratejisiyle çözebiliriz.
 Üçüncü turda Ali kaybetmiş. Oyunun sonunda Esen ve Huriye'nin 24 lirası varsa demekki 3. turdan önce Esen'in'de, Huriye'ninde 12 lirası vardı. Ali'nin ise 3. turdan önce 48 lirası vardı. 2. turda Huriye kaybetmiş. Demekki Huriye'nin 2. oyun'dan önce 42 lirası vermiş, bunun 6'sını Esen'e vermiş aruntı: 12 olmuş. 24'ün içinde Ali'ye vermişti Ali'ninki 48 olmuş. 1. turda Esen kaybetmiş demekki esen Huriye'ye 21, Ali'ye 12 lira vermiş. Esen kaybeden önce Huriye'nin 21, Esen'in 39, Ali'nin 12 lirası vardır. (Geri)

Figure 1. Working backwards (Correct solution-Correct strategy- Indicated strategy correctly)

According to the teacher candidates' solution in Figure 1, she used working backwards strategy and found the correct answer. Her solution is: “We can solve this problem with working backwards strategy. Ali loses in round 3 and gives Esen and Huriye as much money as they each have. At the end of the game Esen has 24 TL and Huriye has 24 TL. This means before round 3 Esen and Huriye has 12 TL, and Ali has 48 TL. Huriye lost in the second round. This means before round 2 Huriye has 42 TL (She gave 6TL to Esen and 24 TL to Ali). In round 1 Esen lost so she gave 21 TL to Huriye and 12 TL to Ali. From that Huriye has 21 TL, Esen has 39 TL and Ali has 12 TL at first.” It was a due that she could understand what the problem is and chose an appropriate strategy for problem solving. She also applied plan and reached the correct answer. She was aware of the strategy what she used. Other teacher candidate tried to solve problem but he could not reach the correct answer. Because he could not choose an appropriate strategy so problem solving process did not finalize successfully. Example of choosing wrong strategy and finding wrong answer was given in Figure 2:

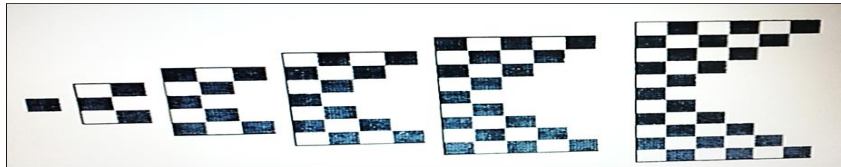
3 tur			
Huriye	Esen	Ali	
$10x$	$10y$	$10z$	$10(y-x-z) = 24$
$10x$	$10(y-x-z)$	$10z$	$y-x-z = \frac{24}{10}$
$10x - 10(y-x-z) - 10z$	$10(y-x-z)$	$10z$	$\rightarrow 1. Tur$
$10(x-z) - 10(y-x-z) - 10z$	$10(y-x-z)$	$10z - [10(x-z) - 10(y-x-z) - 10z]$	$\rightarrow 2. Tur$
-24	$= 24$	$= 24$	

Figure 2. Working backwards (Wrong solution- Wrong strategy)

Teacher candidates did not work back so it was difficult and complex to solve problems and he did not find the correct solution. In other words, mathematics teacher candidates have limited abilities about “planning”. It was a due of they could not choose an appropriate strategy for solving problem.

3.2. Results about Finding Pattern Strategy

Second problem is “The first 6 terms in a sequence are shown in Figure. If the sequence continues in this manner, how many squares will there be in the 10th term and how many of these squares will be shaded?”



It is related to the Finding Pattern Strategy. According to the Table 1, while 59.7% of the teacher candidates solved the problem correctly. 54.9% of them indicated strategy correctly, 1% were wrongly, 3.8% of them did not indicated any strategy. Another finding is that 29.8% of teacher candidates solved the problem wrongly. 18.3% of them did not find the correct answer despite using correct strategy. It was an evidence that, these teacher candidates have lack of knowledge about “applying plan”. 11.5% of them did not reach the correct answer because they used the wrong strategy and 10.5% did not make any solution to the problem. One of the teacher candidates reached the correct answer by applying correct strategy. She found pattern about the shaded squares. She applied finding pattern strategy. Example about correct strategy and correct solution category was given in Figure 3:

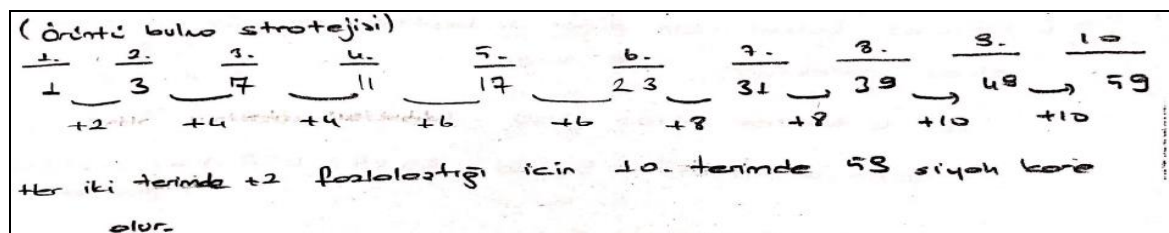


Figure 3. Finding pattern (Correct solution-Correct strategy- Indicated strategy correctly)

According to the solution as seen in Figure 3, she stated that she used finding pattern strategy in problem solving. “In first patterns there is 1 shaded square, second 3, third 7, fourth 11 and fifth 17,.... Each pattern increases by 2, 4, 4, 6, 6, 8, 8, 10 respectively. According to this pattern there will be 59 shaded squares.” According to the result, she chose correct strategy and found the correct solution. Another teacher candidate used finding pattern strategy but she did not reach the correct answer. Example of correct strategy and wrong solution category was given in Figure 4:

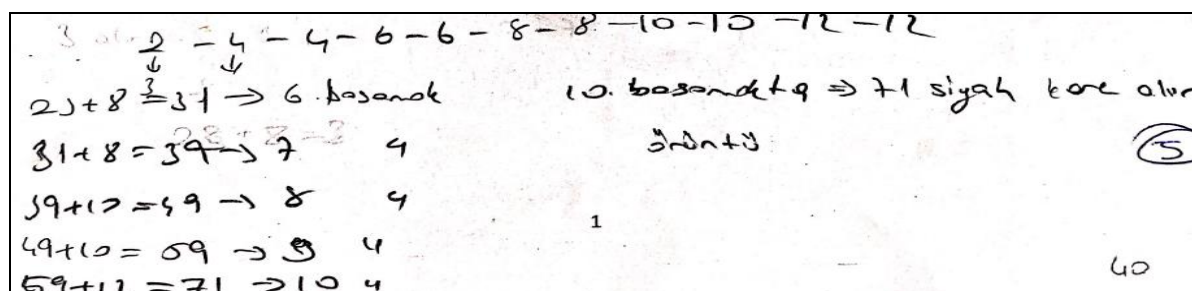


Figure 4. Finding pattern (Wrong solution-Correct strategy)

She stated that she used finding pattern strategy in her problem solving. “Each pattern increases by 2, 4, 4, 6, 6, 8, 8, 10, 10, 12, 12 respectively. In sixth step, there are 31 shaded, in seventh step 39,According to this pattern there will be 71 shaded squares in tenth step.” In this solution teacher candidates chose correct strategy (finding pattern strategy) but she made a mistake during the applying plan. She found the pattern correctly. If she was more careful when counting shaded squares, she could get the correct result.

3.3. Results about Adopting a Different Point of View Strategy

Third problem is “A box contains four slips of paper. On each slip is written one of the four digits 3, 5, 7 and 9. The slips are drawn from the box one at a time. As they are drawn, they are placed from left to right to form a four-digit number. What is the probability that the number formed is a prime number?” This problem is related to adopting a different point of view strategy. According to the Table 1, 42.3% of the teacher candidates solved the problem correctly. 35.6% of them indicated strategy correctly, 4.8% of them indicated wrongly; 1.9% did not indicate any strategy. 36.5% of teacher candidates solved the problem wrongly. 1.9% did not find the correct answer despite using the correct strategy. It was an evidence that these teacher candidates have lack of knowledge about “applying plan”. 34.6% of them did not reach the correct answer because they used the wrong strategy and 21.2% did not make any solution to the problem.

It is a best and simple way to solve this problem is using adopting a different point of view strategy. One of the teacher candidates’ solution which was classified as “Correct Solution-Correct Strategy-Indicated Strategy Correctly” was given in Figure 5:

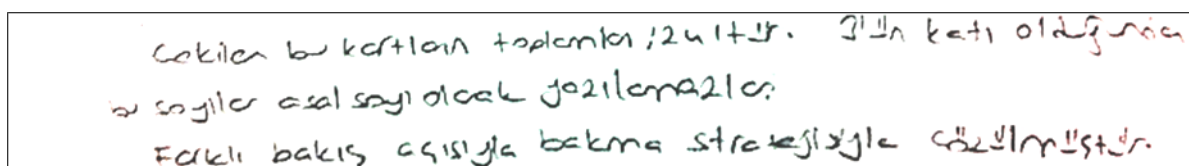


Figure 5. Adopting a different point of view (Correct solution-Correct strategy- Indicated strategy correctly)

As seen in Figure 5, in this problem it is required to look at the problem from different perspective. He stated that “The sum of the numbers is 24. Since the sum of these numbers is 3 or multiple of 3. The probability of being prime number is zero. I used adopting a different point of view strategy.”. It was solved by adopting a different point of view strategy. It was a due that he could understand what the problem was and chose an appropriate strategy for solving problem. She also applied plan and reached the correct answer. She was aware of strategy what she used.

This problem also solved by forming four digit numbers. One of the teacher candidate tried to find correct solution by this strategy. Another solution which was classified as wrong solution was given in Figure 6:

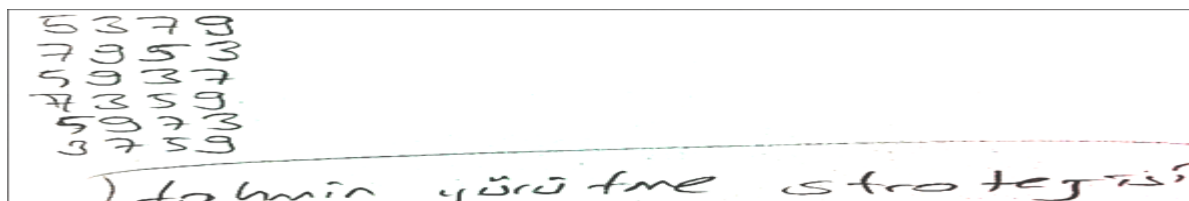


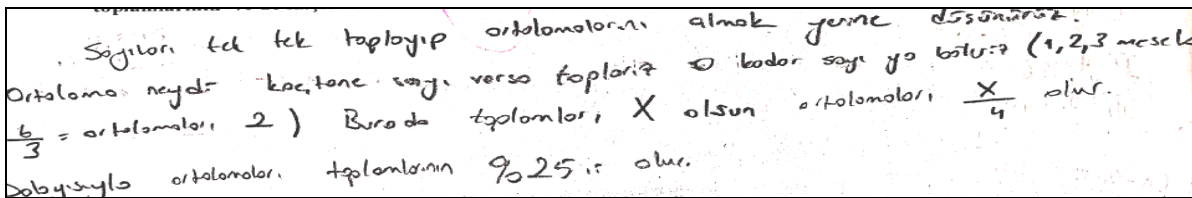
Figure 6. Adopting a different point of view (Wrong solution-Wrong strategy)

According to the teacher candidates’ solution, he stated that he used intelligent guessing strategy, he formed four digit numbers but he did not aware of the sum of numbers were multiples of three. He did not find the correct answer so it was classified as wrong solution and wrong strategy.

3.4. Results about Solving a Simpler Analogous Problem Strategy

Fourth problem is "Given the four numbers 7.895, 13.127, 51.873 and 7.356. What percent of their sum is their average?" (Solving a Simpler Analogous Problem). Problem is about solving a simpler analogous problem strategy. According to the Table 1, 73.1% of the teacher candidates solved the problem correctly. 27.9% of them indicated strategy correctly, 2.9% of them indicated strategy wrongly and 42.3% did not indicate any strategy. 16.4% of teacher candidates solved the problem wrongly. 1% did not find the correct answer while using the correct strategy. It was an evidence that these teacher candidates have lack of knowledge about "applying plan". 15.4% of them did not reach the correct answer because they used the wrong strategy and 10.5% did not make any solution to the problem.

Also this problem can be solved by finding the sum of numbers and average of numbers. But changing the given problem into one that may be easier to solve is best way to find correct answer. Teacher candidates thought simply and found the correct solution by using correct strategy. One of the teacher candidates' solution about "Correct Solution-Correct Strategy-Indicated Strategy Correctly" category was given in Figure 7:



Sayıları tek tek toplayıp ortalamaları almak yerine düşünürüz.
Ortalama nedir? kaç tane sayı varsa topları? 3 kadar sayı ya biliriz (1,2,3 mesela)
 $\frac{6}{3} = \text{ortalama}$ 2) Burada topları X olsun ortalaması $\frac{X}{4}$ olur.
Dolayısıyla ortalamaları toplamının %25'i olur.

Figure 7. Solving a simpler analogous problem (Correct solution-Correct strategy- Not indicated strategy)

According to the Figure 7, she stated that "Instead of adding numbers one by one and calculating the average, we think that, what an average is. Add the given numbers and divide count of numbers. (For example, numbers are 1, 2, 3; their average is $6/3=2$). The sum of the given numbers is X; their average is $X/4$ so 25 percent of their sum, is their average." It was a due that she could understand what the problem was and chose an appropriate strategy for solving problem. She also applied plan and reached the correct answer. She was aware of the strategy what she used.

One of the teacher candidate used correct strategy but she made a mistake. There were four different numbers. She should have divided the sum of numbers by four instead of three. Another solution of problem was given in Figure 8:



$\left(\frac{a+b+c}{3}\right) = \frac{a+b+c}{\text{toplama}}$
toplama toplama
toplam 3 kat fazlaysa
%33,3%

Figure 8. Solving a simpler analogous problem (Wrong solution-Correct strategy)

As seen in Figure 8, he used solving a simpler analogous problem strategy but he could not find the correct solution. According to the solution he found the sum of three numbers. After that he divided the sum by three to find mean. "The sum of number $a+b+c$. The mean of these numbers $(a+b+c)/3$. That is 33.3%"

3.5. Results about Considering Extreme Cases Strategy

Fifth problem is "Ayşe's mathematics teacher has given the class five tests this semester, each scored from 0 to 100 inclusive. Ayşe had an average of 90 for five tests. What was the lowest possible score Ayşe could have earned on any one test?" (Considering Extreme Cases). According to the Table 1,

86.5% of the teacher candidates solved the problem correctly. 81.7% of them indicated strategy correctly, 4.8% of them indicated strategy wrongly. In addition, 11.5% of teacher candidates solved the problem wrongly. 7.7% of them did not find the correct answer while using the correct strategy. It was an evidence that teacher candidates have lack of knowledge about “applying plan”. 3.8% of them did not reach the correct answer because they used the wrong strategy and 1.9% did not make any solution to the problem. One of the teacher candidate solution was given in Figure 9:

Bir testten aldığı puanın en az olabilmesi için, diğer dört testten aldığı puanın en çok olması gerekir.

$$\frac{x + 100 + 100 + 100 + 100}{5} = 90 \quad x + 400 = 450 \quad \text{Bir testten en az 50 puan alınır.}$$

$$x = 50$$

• Gerçeğe doğru işlem yapma stratejisi ("Hangi sayının kaç farkının site'ri 90'dır?" diye düşünerek)

Figure 9. Considering extreme cases (Correct solution-Correct strategy-Indicated strategy wrongly)

This problem is related to considering extreme cases strategy. According to the solution in Figure 9 : “In order to have a minimum score from a test, the score from other four tests must be the maximum. Four test must be 100 so the sum of four test 400, so minimum score is 50.” He found the correct answer by using considering extreme cases strategy so it was classified as correct solution, correct strategy. He also indicated that “I solved by using working backwards strategy” so he indicated strategy wrongly. Another solution which was classified as “Wrong Solution-Wrong Strategy” was given in Figure 10:

100 100 100 100 100 olduğu için ort. 90 olurdu.

$$\frac{100 + 100 + 100 + 100 + 100}{5} = 90 \text{ dir.}$$

olduğu notların toplamı = 450 olmalı. hepsinde 90 olmalı olur. en az 90 bulmak için $89 \times 5 = 445$ buluruz. $450 - 445 = 5$ en az olabilir.

Figure 10. Considering extreme cases (Wrong solution-Wrong strategy)

As seen in Figure 10, solution of problem is: “Ayşe had an average of 90 for five tests so the sum of her tests were 450. She gets 90 of them all. To find the least, $89 \times 5 = 445$ and $450 - 445 = 5$ ”. This solution is wrong because he used wrong strategy.

3.6. Results about Making Drawing Strategy

Sixth problem is “Hasan and Ahmet are both part-time workers in the local pizza shop. The shop is open seven days a week. Hasan works one day and then has two days off before he works again. Ahmet works one day and then has three days off before he works again. Hasan and Ahmet both worked on Tuesday, March 1st. On which other days in March do Hasan and Ahmet work together?” (Making Drawing). According to the Table 1, while 81.7% of the teacher candidates solved the problem correctly. 64.4% of them indicated strategy correctly, 17.3% were wrongly. 13.5% of teacher candidates solved the problem wrongly. 5.8% did not find the correct answer while using the correct strategy. 7.7% of them did not reach the correct answer because they used the wrong strategy and 4.8% did not make any solution to the problem. One of the correct solution was given in Figure 11:

H = Hasan A = Ahmet

1 Mart 2 Mart 3 Mart 4 Mart 5 Mart 6 Mart

	A, H			H	A	
	H		A	H		
13 Mart Salı	A, H			H	A	
	H		A	H		
25 Mart Pazar	A, H			H	A	

13 Mart Salı } Birlikte çalıştıkları günler.
25 Mart Pazar }

Tablo (Form) Çizme Stratejisi

Figure 11. Making drawing (Correct solution-Correct strategy- Indicated strategy correctly)

As seen in Figure 11, he made a time table for solution and he stated that he used making drawing (table) strategy and found Hasan and Ahmet work together on 13 and 25 March. He also indicated that he used "Making Drawings Strategy" so it was classified as "Correct Solution-Correct Strategy- Indicated Strategy Correctly". Another solution which was classified as "Wrong Solution-Correct Strategy" was given in Figure 12:

1 Mart Pazar	3 Cuma	3 Cumartesi	4 Pazar	5 Pazartesi	Salı	Çarşamba
Birlikte çalıştıkları			H	A		H
H						
A						

1, 4, 7, 10, 13, 16, 19, 22, 25, 28
14, 15, 18, 19, 24, 29

19 Mart Salı

Figure 12. Making drawing (Wrong solution-Correct strategy)

According to the Figure 12, she did not find correct answer because she did not understand the problem. She solved problem as "Hasan works one day and then has three days off before he works again and Ahmet works one day and then has four days off before he works again. According to this Hasan works on 1, 4, 7, 10, 13, 16, 19, 22, 25, 28 March and Ahmet works on 1, 5, 9, 14, 19, 24, 29 March. She found that they work together on 19 March."

3.7. Results about Intelligent Guessing and Testing Strategy

Seventh problem is "Emine and Emel make bracelets from beads and sell them at local crafts shown. Yesterday they sold some of the bracelets for 1 TL each and half as many for 1.50 TL each. Altogether they took in 87.50 TL. How many of each type did they sell?" (Intelligent Guessing and Testing). According to the Table 1, while 90.4% of the teacher candidates solved the problem correctly. 85.6% of them indicated strategy correctly, 4.8% of them indicated strategy wrongly. 7.7% of teacher candidates solved the problem wrongly. 2.9% did not find the correct answer while using the correct strategy. It was an evidence that these teacher candidates have lack of knowledge about "applying plan". 4.8% of them did not reach the correct answer because they used the wrong strategy and 1.9% did not make any solution to the problem. One of the solution was given in Figure 13:

1,5 liralik	1 liralik	Toplam
20	40	$30 + 40 = 70 \times$
23	46	$34,5 + 46 = 80,5 \times$
25	50	$37,5 + 50 = 87,5 \checkmark$

Tahmin ve Kontrol

Figure 13. Intelligent guessing and testing (Correct solution-Correct strategy- Indicated strategy correctly)

As seen in Figure 13, she found the correct answer by using intelligent guessing and testing strategy. She guessed the solution and test to show it is correct or not. She indicated that “I used intelligent guessing and testing strategy” so it was classified as “Correct Solution-Correct Strategy- Indicated Strategy Correctly.” Another teacher candidate used the same strategy but he did not find correct answer. Solution of the problem which was classified as “Wrong Solution-Correct Strategy” was given in Figure 14:

1 Liralik Bireklik	15 Liralik Bireklik	Toplam
20 tane = 20 Lira	20 tane = 30 Lira	50 Lira
30 tane = 30 Lira	30 tane = 45 Lira	85 Lira
60 tane = 60 Lira	60 tane = 60 Lira	100 Lira
75 tane = 75 Lira	75 tane = 72,5 Lira	85 Lira
75'ler tane satmislardir.		Tahmin ve Kontrol

Figure 14. Intelligent guessing and testing (Wrong solution-Correct strategy)

According to the solution that was given in Figure 14, he indicated that he used intelligent guessing and testing. Despite using the correct strategy, he could not reach the correct result. Because he made a mistake on addition of numbers in applying plan stage.

3.8. Results about Accounting for All Possibilities Strategy

Eight problem is “Ahmet invited 17 friends to a dinner party at her house last Friday evening. He gave each guest a card with a number form 2 through 18, reserving number 1 for himself. When he had everyone paired off at the dinner table, he noticed that the sum of each couple’s numbers was a perfect square. What number did Ahmet’s partner have?” (Accounting for All Possibilities). According to the Table 1, while 46.1% of the teacher candidates solved the problem correctly. 42.3% of them indicated strategy correctly, 3.8% were wrongly. 44.3% of teacher candidates solved the problem wrongly. 1% did not find the correct answer while using the correct strategy. 43.3% of them did not reach the correct answer because they used the wrong strategy and 9.6% did not make any solution to the problem. One of the solution which was classified as “Correct Solution-Correct Strategy-Indicated Strategy Correctly” was given in Figure 15:

16 kişi	8 çift (Ahmet hariç)
1. çift = $2+14=16$	2 3 4 5 6
2. çift = $10+6=16$	
3. çift = $17+8=25$	
4. " = $4+12=16$	
5. " = $9+16=25$	
6. " = $5+11=16$	
7. " = $13+3=16$	
8. " = $18+7=25$	
Ali ve kız arkadaşısı = $1+15=16$	
Ali'nin kız arkadaşının numarası = <u>15</u>	
Tahmin ve Kontrol	

Figure 15. Accounting for all possibilities (Correct solution-Correct strategy- Indicated strategy correctly)

According to the solution that was given in Figure 15, teacher candidate found the correct answer by using intelligent guessing and testing strategy. “First couple 2 and 14. $2+14=16$, second couple $10+6=16$,... Ahmet’s partner is number 15.” She found the pair of numbers that the sum of them

were perfect square. Another solution which was teacher candidate tried to find correct answer was given in Figure 16:

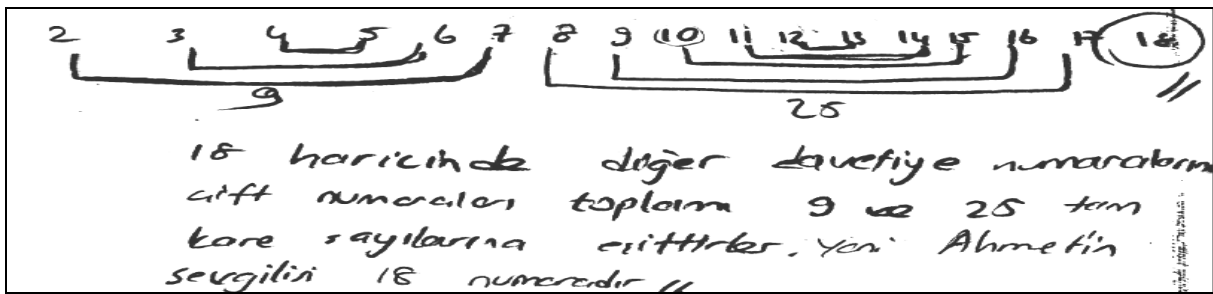


Figure 16. Accounting for all possibilities (Wrong solution-Wrong strategy)

According to the Figure 16, it was seen that teacher candidate used correct strategy but he did not find correct answer. He also tried to find the pair of numbers that the sum of them were perfect square but he did not. "The sum of numbers except 18 is 9 and 25 so Ahmet's partner is 18."

3.9. Results about Organizing Data Strategy

Ninth problem is "Find the sum of the terms in the series $20^2 - 19^2 + 18^2 - 17^2 + 16^2 - 15^2 + \dots + 4^2 - 3^2 + 2^2 - 1^2$." (Organizing Data). According to the Table 1, while 51% of the teacher candidates solved the problem correctly. 50% of them indicated strategy correctly, 1% were wrongly. 34.6% of teacher candidates solved the problem wrongly. 18.3% did not find the correct answer while using the correct strategy. It was an evidence that these teacher candidates have lack of knowledge about "applying plan". 16.3% of them did not reach the correct answer because they used the wrong strategy and 14.4% did not make any solution to the problem. One of the teacher candidate solution was given in Figure 17:

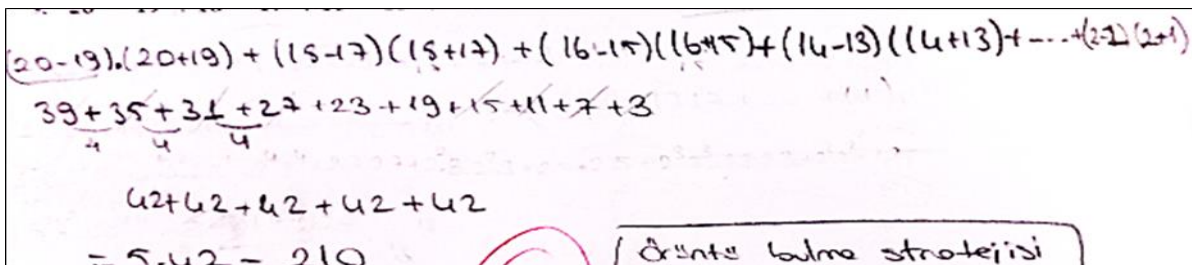


Figure 17. Organizing data (Correct solution- Correct strategy- Indicated strategy correctly)

When the teacher candidates' solution was examined, it was seen that teacher candidate found a pattern and solved problem correctly by using finding pattern strategy. Another solution which was classified as "Wrong Solution- Correct Strategy" was given below:

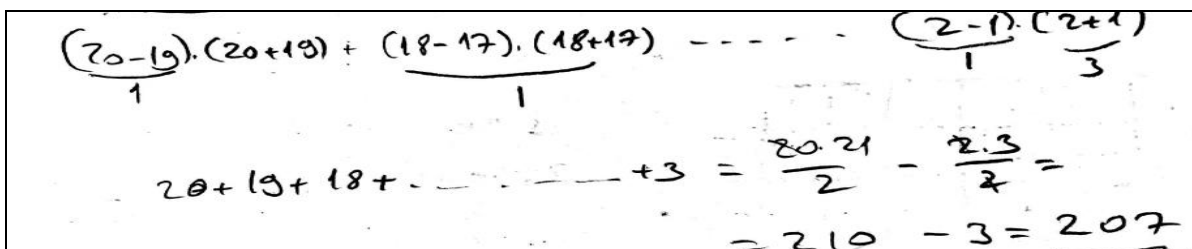


Figure 18. Organizing data (Wrong solution- Correct strategy)

As seen in Figure 18, teacher candidate tried to find correct answer by organizing data. He used correct strategy but he did not find correct answer. Because he made a mistake during the applying plan.

3.10. Results about Logical Reasoning Strategy

Tenth problem is “Hidayet is driving to the local shopping mall, a distance of 2 miles. He drives the first mile at exactly 30 mph then realizes that he will be late for his appointment and decides to drive faster. At what speed must he drive the second mile to average 60 mph for the 2 miles?”. According to the Table 1, while 4.8% of the teacher candidates solved the problem correctly. 3.8% of them indicated strategy correctly and 1% did not indicate any strategy. 82.7% of teacher candidates solved the problem wrongly. 1% did not find the correct answer while using the correct strategy. It was an evidence that these teacher candidates have lack of knowledge about “applying plan”. 81.7% of them did not reach the correct answer because they used the wrong strategy and 12.5% did not make any solution to the problem. One of the teacher candidate solution was given in Figure 19:

30 km	1 saat
1 km	2 dk

60 km	1 saat
2 km	2 dk

Eğer bunu düz mantık ile çözmek istesek 60km gora basittir
 $\frac{1 \cdot 30 + 90 \cdot 1}{2} = 60$ km ortama olması için hızını 90 km çıkarma
 Si gerektiğini aktarırdık ancak olaya matematiksel açıdan
 bakalım, 2 km yolu 60 km ortalama hızla giden araç gideceği yolu
 2 dk alır. Biz verilen soruda araç 1 km, 30 km hızla aldığı için her
 2 dk 90m 2 dk. dir yon randevuya yetişememektedir.

Figure 19. Logical reasoning (Correct solution-Correct strategy-Not indicated strategy)

According to the Figure 19, teacher candidate solved as “If we solve this question with a straightforward logic, we can say that the speed should be 90 km so that $(1 \cdot 30 + 90 \cdot 1) / 2 = 60$ is the average. However, from a mathematical point of view, the 2 km road, the vehicle with an average speed of 60 km / h, takes 2 minutes to get there. In our question, it takes 2 minutes to get 1 km at 30 km/h. He can not make it to his appointment.” Another solution was given in Figure 20:

$x = v \cdot t$
 $1 = 30t$

$\frac{30 + a}{2} = 60$ $30 + a = 120$
 $a = 90$

Figure 20. Logical reasoning (Wrong solution-Wrong strategy)

As seen in Figure 20, he used solving equation and tried to find solution using average speed of the car but he did not find correct answer. Because in this problem it is needed to think logically. For that reasons problem solution was classified as “Wrong Solution-Wrong Strategy”. Another solution which was classified as “Wrong Solution-Correct Strategy” was given in Figure 21.

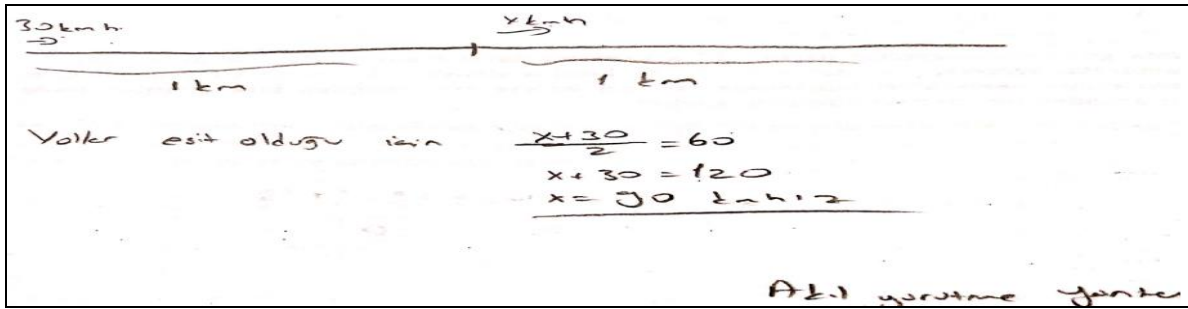


Figure 21. Logical Reasoning (Wrong Solution-Correct Strategy)

According to the solution in Figure 21, teacher candidate used logical reasoning strategy however he did not find correct answer. " $(x+30)/2=60$, $x+30=120$, $x=90 \text{ km/h}$. He tried to use correct strategy but he looks problem with straightforward logic so it was classified as "Wrong Solution-Correct Strategy".

4. Results and Discussion

The aim of the study was to examine mathematics teacher candidates' strategies used in solving non-routine problems. In this context, it has been determined the teacher candidates' strategies used in solving non-routine problems and whether they are aware of the strategies they use in solving the given problems or not. According to the findings, the problem solving performances of the teacher candidates are not at high level. Gürbüz and Güder (2016) also argued that teachers are proficient in solving problems but are incompetent in employing different strategies. The more pedagogical knowledge the teacher has on problem solving, the more effective teaching he/she has done in the classroom (Türker-Biber, Aylar, Sonay-Ay & Akkuş-İspir, 2017). For this reason, teacher candidates should be encounter environments where they use different strategies and can create new problems (MoNE, 2013; Van De Walle, Karp & Williams, 2012).

According to the findings teacher candidates were most successful in intelligent guessing and testing, considering extreme cases and making drawing strategy. This is the reason why the teacher candidates used these strategies successfully that they have been used that strategies since primary school. Larkin and Simon (1987) stated that diagrams can group all information that is used together, thus it avoids searching for a large amount of elements necessary for problem solving deduction. They were successful in solving a simpler analogous problem, finding a pattern, organizing data, accounting for all possibilities, adopting a different point of view and working backwards strategy respectively. The strategies that students and teacher candidates use and succeed in problem solving differ in researches. Gökkurt-Özdemir, Koçak and Soylu (2018) found that preservice middle school mathematics teachers usually used trial and error (intelligent guessing and testing) strategy. Altun and Arslan (2006) revealed that 7th and 8th grade students were most successful in using systematic listing, making drawing, and working backwards strategies. Çelebioğlu, Yazgan and Ezentaş (2010) indicated that the strategy used most successfully by the first graders is look for a pattern. Yazgan and Bintaş (2005) found that fourth and fifth grade students mostly use intelligent guessing and testing strategies; least finding pattern, working backwards and making drawing. Arslan and Yazgan (2015) stated that, students were comfortable in using "look for a pattern" and "make a drawing" strategies and the most unfavorable strategy for them was "simplify the problem". Additionally, there were enterprises to use "write an equation" strategy. On the other hand, Yazgan (2015) investigated the role of each strategy in explaining sixth graders' (12-13 years old students') non-routine problem solving success. According to the findings, even though almost all strategies had statistically significant roles, look for a pattern and make a drawing strategies were conspicuous to explain non-routine problem solving performance. From this findings, it can be argued that students and teacher candidates were successful in different problem solving strategies.

The problem which teacher candidates were least successful is about the logical reasoning strategy. This problem requires higher level thinking. Unlike other problems, this problem has no answer that can be expressed by numbers. Teacher candidates try to find a number. This situation can be explained by the tendency of teacher candidates to find a more numerical solution in mathematics lessons (Kılıç,

2011). Many students tended to apply the necessary procedures to the given numbers and find solution instead of cognitive activities such as judging the solution process, analyzing the problem, evaluating the results (Arslan & Altun, 2007).

The result of this study indicated that some of mathematics teacher candidates had adequate problem solving skills. They found correct answer by using correct strategies. During this period, they used Polya's problem solving steps. They understood the problem, made a plan, applied plan and checked the answer whether it was correct or not. Some of teacher candidates could not solve problems correctly because they used the wrong strategy. In other words, mathematics teacher candidates had limited abilities at the stage of "understanding of a problem" and "planning". In some situations, teacher candidates often tried to solve the problem by solving equations rather than using appropriate strategies and that makes the process difficult and complicated. It was a due of they could not understand what the problem says and they could not choose an appropriate strategy for problem solving. Using appropriate problem solving strategy is important in terms of being successful in problem solving (Ersoy & Güner, 2014). Yew and Zamri (2016) also stated that pre- service teachers who were unable to solve the problem correctly seemed to use limited and incorrect mathematical terminology, lack understanding of the problem, were unable to make representations of the word problems, lack some basic knowledge. Furthermore, Abdullah, Rahman and Hamzah (2017) indicated that students have lack of planning, implementation, and revision skills during the problem solving.

Some of teacher candidates could not reach the correct answer, even though they used correct strategies in solving problems. It was an evidence that teacher candidates have lack of knowledge about "applying plan". This situation is the result of teacher candidates behave carelessly while performing the plan and making operations. On the other hand, some of teacher candidates could solve problems correctly but they could not indicate strategy correctly. It can be said that some of the mathematics teacher candidates were not aware of the strategies they used in problem solving. Çeker and Ev-Çimen (2017) also indicated that teachers often use problem solving strategies in their lessons in an unplanned way, but most teachers do not have the theoretical background of the strategies.

From the results of the study, teacher educators should teach problem solving strategies effectively in their method course. Teacher candidates should be encounter different types of non-routine problems in their teacher education programme. Teacher educators should also consider new approaches such as problem posing for learning problem-solving strategies in their method course (Kılıç, 2017). Problem solving process of teacher candidates should be researched by longitudinal studies.

References

- [1] Abdullah, A. H., Rahman, S. N. S. A., & Hamzah, M. H. (2017). Metacognitive skills of malaysian students in non-routine mathematical problem solving. *Bolema: Boletim de Educação Matemática*, 31(57), 310-322.
- [2] Aksoy, Y., Bayazit, İ., & Kırnay- Dönmez, S. M. (2015). Prospective primary school teachers' proficiencies in solving real-world problems: Approaches, strategies and models. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(4), 827-839.
- [3] Altun, M. (2008). *İlköğretim ikinci kademe (6, 7 ve 8. sınıflarda) matematik öğretimi*. Bursa: Aktuel Yayıncılık.
- [4] Altun, M., & Arslan, Ç. (2006). İlköğretim öğrencilerinin problem çözme stratejilerini öğrenmeleri üzerine bir çalışma. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi*, 19(1), 1-21.
- [5] Apostol, E. M. D. (2017). Problem solving heuristics on non-routine problems of college students. *American Journal of Educational Research*, 5 (3), 338-343. doi: 10.12691/education-5-3-16.
- [6] Arslan, Ç., & Altun, M. (2007). Learning to solve non-routine problems. *Elementary Education Online*, 6(1), 50-61.
- [7] Arslan, C., & Yazgan, Y. (2015). Common and flexible use of mathematical non routine problem solving strategies. *American Journal of Educational Research*, 3(12), 1519-1523.

- [8] Baykul, Y. (2009). *İlköğretimde matematik öğretimi (6-8. Sınıflar)*. Ankara: Pegem A Yayıncılık.
- [9] Che, M., Wiegert, E., & Threlkeld, K. (2012). Problem solving strategies of girls and boys in single-sex mathematics classrooms. *Educational Studies in Mathematics*, 79(2), 311-326.
- [10] Çelebioğlu, B., & Yazgan, Y. (2009). İlköğretim öğrencilerinin bağıntı bulma ve sistematik liste yapma stratejilerini kullanma düzeyleri. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi*, XXII (1), 15-28.
- [11] Çelebioğlu, B., Yazgan, Y., & Ezentaş, R. (2010). Usage of non-routine problem solving strategies at first grade level. *Procedia-Social and Behavioral Sciences*, 2(2), 2968-2974.
- [12] Çeker, F., & Çimen, E. E. (2017). Ortaokul matematik öğretmenlerinin problem çözme stratejilerine ilişkin görüşleri. *Türk Dünyası Uygulama ve Araştırma Merkezi Eğitim Dergisi*, 2(1), 44-60.
- [13] Elia, I., Heuvel-Panhuizen, M., & Kolovou, A. (2009). Exploring strategy use and strategy flexibility in nonroutine problem solving by primary school high achievers in mathematics. *ZDM Mathematics Education*, 41, 605-618.
- [14] Ersoy E., & Güner P. (2014). Matematik öğretimi ve matematiksel düşünme. *Journal of Research in Education and Teaching*, 3, 102-112.
- [15] Fraenkel, J. R., & Wallen, N. E. (2005). *How to design and evaluate research in education* (3rd edition). New York: McGraw-Hill.
- [16] Gökkurt-Özdemir, B., Erdem, E., Örnek, T., & Soylu, Y. (2018). Are middle school mathematics teachers able to solve word problems without using variable?. *International Journal of Mathematical Education in Science and Technology*, 49(1), 85-106.
- [17] Gökkurt-Özdemir, B., Koçak, M., & Soylu, Y. (2018). Ortaokul matematik öğretmeni adaylarının sözel problemleri deęişkensiz çözümede kullandıkları stratejiler ve yöntemler. *Trakya Üniversitesi Eğitim Fakültesi Dergisi*, 8(3), 449-467.
- [18] Gür, H., & Hangül, T. (2015). Ortaokul öğrencilerinin problem çözme stratejileri üzerine bir çalışma. *Pegem Eğitim ve Öğretim Dergisi*, 5 (1), 95-112. Retrieved August 10, 2017 from <http://dergipark.gov.tr/download/article-file/209568>.
- [19] Gürbüz, R., & Güder, Y. (2016). Matematik öğretmenlerinin problem çözümede kullandıkları stratejiler. *Journal of Kirsehir Education Faculty*, 17(2), 371-386.
- [20] Inoue, N. (2005). The realistic reasons behind unrealistic solutions: The role of interpretive activity in word problem solving. *Learning and Instruction*, 15, 69-83.
- [21] Kılıç, Ç. (2011). İlköğretim Matematik öğretmen adaylarının standart olmayan sözel problemlere verdikleri yanıtlar ve yorumlar. *Ahi Evran Üniversitesi Eğitim Fakültesi Dergisi*, 12(3), 55-74.
- [22] Kılıç, Ç. (2017). A new problem-posing approach based on problem-solving strategy: Analyzing pre-service primary school teachers' performance. *Educational Sciences: Theory & Practice*, 17, 771-789. <http://dx.doi.org/10.12738/estp.2017.3.0017>
- [23] Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, 11, 65-99.
- [24] Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis*. Thousand Oaks (CA): Sage Publications.
- [25] MoNE (2006). *İlköğretim (6-8) Matematik öğretim programı*. Ankara: MEB Yayınları. MoNE (2013). Ortaokul Matematik Dersi (5, 6,7 ve 8. Sınıflar) Öğretim Programı. Ankara: M.E.B. Retrieved 10 April, 2014 from <http://ttkb.meb.gov.tr/program2.aspx?islem=2&kno=215>.
- [26] National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

- [27] Olkun, S., Şahin, Ö., Akkurt, Z., Dikkartın, F. T., & Gülbağcı, H. (2009). Problem solving and generalization through modeling: A study on elementary school students. *Education and Science*, 34 (151), 65-73.
- [28] PISA (2005). *PISA 2003 araştırması ulusal nihai raporu*. Retrieved January 30, 2017 from <http://pisa.meb.gov.tr/wp-content/uploads/2013/07/PISA-2003-Ulusal-Nihai-Rapor.pdf>.
- [29] Polya, G. (1962). *Mathematical discovery: On understanding, learning, and teaching problem solving*. New York: John Wiley.
- [30] Polya, G. (1990). *How to solve it, a new aspect of mathematical method*, Penguin Books, Londra. Feryal Halatçı, Trans. (1997), Sistem Yayıncılık, İstanbul.
- [31] Posamentier, A. S., & Krulik, S. (1998). *Problem solving strategies for efficient and elegant solutions: A resource for the mathematics teachers*. USA: Corwin Press Inc. Schoenfeld.
- [32] Ross, B.H., & Kennedy, P.T. (1990). Generalizing from the use of earlier examples in problem solving. *Journal of Experimental Psychology*, 16(1), 42-45.
- [33] Schunk, D. H. (2009). *Öğrenme teorileri eğitimsel bir bakışla* (M. Şahin, Trans.). Ankara: Nobel Yayın Dağıtım.
- [34] Sheffield, L. J., & Cruikshank, D. E. (2005). *Teaching and learning mathematics, Pre-Kindergarten through middle school*. Wiley Jossey Bass Education, USA.
- [35] Türker-Biber, B., Aylar, E., Sonay-Ay, Z., & Akkuş-İspir, O. (2017). İlköğretim matematik öğretmen adaylarının problem çözmeye dair pedagojik alan bilgilerinin sınıf içi gözlem ve görüşme yoluyla belirlenmesi. *Kastamonu Eğitim Dergisi*, 25(4), 1483-1498.
- [36] Ulu, M. (2011). *İlköğretim 5. sınıf öğrencilerinin rutin olmayan problemlerde yaptıkları hataların belirlenmesi ve giderilmesine yönelik bir uygulama*. (Unpublished doctoral dissertation, Gazi University, Turkey). Retrieved from <https://tez.yok.gov.tr/UlusalTezMerkezi/>
- [37] Van De Walle, J.A, Karp, K.S. & Bay-Williams, J.M. (2012). İlkokul ve ortaokul matematiği gelişimsel yaklaşımla öğretim. (Soner Durmuş, Trans.). Ankara: Nobel Yayın Dağıtım. (Original work published in 2010).
- [38] Verschaffel, L., De Corte, E., & Lasure, S. (1994). Realistic considerations in mathematical modelling of school arithmetic word problems. *Learning and Instruction*, 4, 273-294.
- [39] Yazgan, Y. (2015). Sixth graders and non-routine problems: Which strategies are decisive for success?. *Educational Research and Reviews*, 10(13), 1807.
- [40] Yazgan, Y., & Bintaş, J. (2005). İlköğretim dördüncü ve beşinci sınıf öğrencilerinin problem çözme stratejilerini kullanabilme düzeyleri: Bir öğretim deneyi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 28, 210-218.
- [41] Yeo, K. K. J. (2009). Secondary 2 students' difficulties in solving non-routine problems. *International Journal for Mathematics Teaching and Learning*, 8, 1-30.
- [42] Yew, W. T., & Zamri, S. N. A. S. (2016). Problem solving strategies of selected pre-service secondary school mathematics teachers in Malaysia. *Malaysian Online Journal of Educational Sciences*, 4(2), 17-31.

Author

Melihan Ünlü, Faculty of Education, Aksaray University, Aksaray (Turkey). E-mail: melihanunlu@yahoo.com

Acknowledgement

This article is presented as an oral presentation at ISER 42nd International Conference in Zurich, Switzerland on 22nd -23rd November 2016.