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PROBLEMS AND SOLUTION SUGGESTIONS IN TEACHING ROTATION TRANSFORMATION IN CONIC SECTIONS

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Abstract: Circle, ellipse, parabola, hyperbolic curves are defined as conic sections from the intersection of a coni with a plane at different angles. These curves can be found in different positions depending on the different angles. In this case, rotation transformation can be applied to the curves so that their major axes are brought to the horizontal position. The purpose of this study is to identify the problems experienced in using mathematics teacher candidates' rotation transformation in conic sections and to develop solution proposals for these problems. 40 teacher candidates participated in this study. An action plan has been designed and implemented by the researchers in order to solve these problems by starting from the problems experienced by the prospective teachers. At the end of the study, it was determined that the success of the teacher candidates drawing the graphs of the curves was increased.

Key words: conic sections, coordinate transformations, rotation transformation, action research

1. Introduction

Circle, ellipse, parabola, hyperbolic curves which are defined as the conic sections are obtained from the intersection of a cone with a plane at different angles in figure 1.

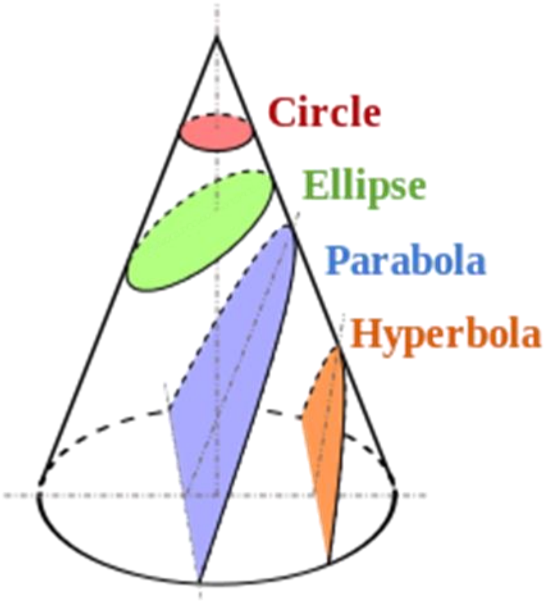

Figure 1. *The intersection of a cone with a plane*

Circle is a conic section obtained by intersection of a plane parallel to the base of cone, ellipse by taking a section with a plane under a certain angle, parabola is obtained by taking section with a plane which is parallel to side of cone, hyperbola is obtained by intersection of a plane parallel to the axis of

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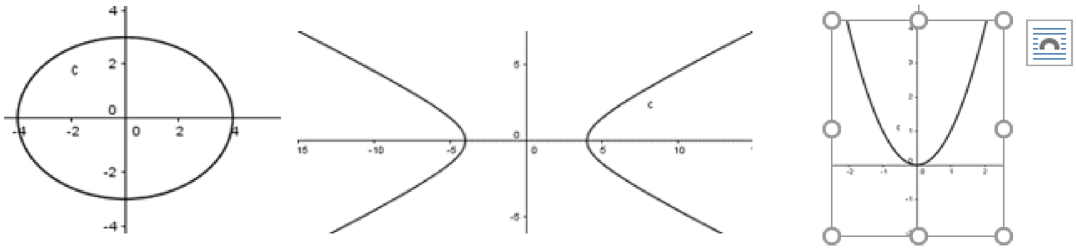
the cone. As these curves are examined analytically, they can be found in different positions (principal axis being as horizontal, vertical or any skewed line.) Algebraic equations based on the positions of these curves can be obtained.

Students learn conic sections as geometric and use them to manipulate as algebraic by finding focal point, vertex and directrix of section of vertex (Jang. & Kang, 2010). Therefore, these curves is taught by using the equations in the standard form (Figure 2).



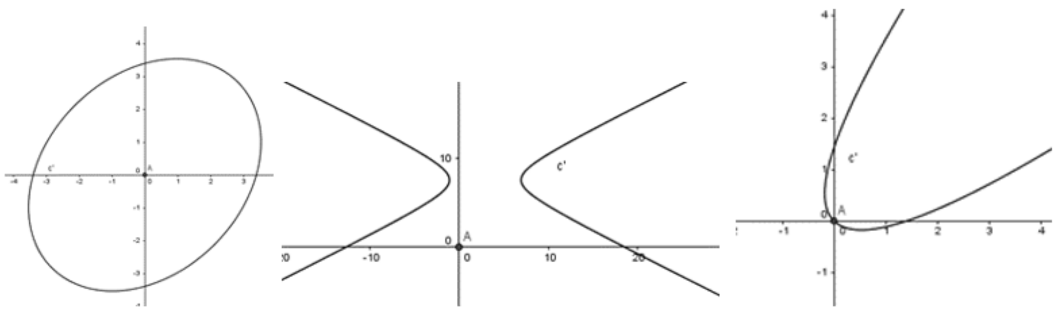
(a) (b) (c)

$\frac{x^2}{16}+\frac{y^2}{4}=1$ $\frac{x^2}{16}-\frac{y^2}{4}=1$ $y=x^2$

Figure 2 (a), (b), (c). *Standard equation and graph in standard form*

If the equations of the curves are not in standard form, they can be standardized with the appropriate transformations of coordinate. In this case, rotation can be applied to the curves by taking their principal axes to the horizontal position. The most common use of these curves in the analytic examination is the standard form chosen the principal axis as *x* axis and the central as origin. Coordinate transformations are changes in position of a shape.

For example, the curves given in Figure 3 are obtained by applying coordinate transformations to the curves in Figure 2 and their positions in the coordinate axes have changed.



(a) (b) (c)

Figure 3 (a), (b), (c). *Graphs of Conic sections*

Translation, rotation and reflection transformations are known coordinate transformations. Translation transformation is the translating of the orthogonal coordinate axes by a certain vector as they are parallel to themselves. Thus, the hyperbolic graph on the standard form in Figure 2b is obtained by applying the translation transformation to the hyperbolic curve in Figure 3b.

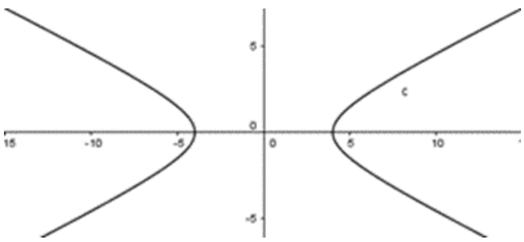


Figure 2b. *The hyperbolic graph on the standard form*

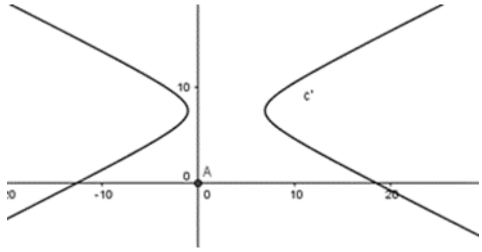


Figure 3b. *Applying the translation transformation to the hyperbolic curve*

The set of points given by the equation (curve) is translated to the given vector to obtain a new image (Figure 3b). Thus, the equation of the curve in standard form can be found by the translation transformation. The learners become able to draw the graph represented by the set of points of the curve. However, translation relation may be not always enough to find the geometric loci of a set of points given algebraic equation. In this case, other coordinate transformations are needed.

Rotation transformation is one of these coordinate transformations. Rotation transformation is defined as the rotation of the x and y axes around an initial point by a specific angle. Thus, position of curve changes around a given point in the plane by a certain angle with rotation transformation. For example, in figure 4, the first shape is rotated clockwise about the origin by 45 degrees to obtain the second image in standard form.

In figure 4, the standard equation of the ellipse given with the algebraic equation is found by rotating 45 degrees clockwise around the origin.

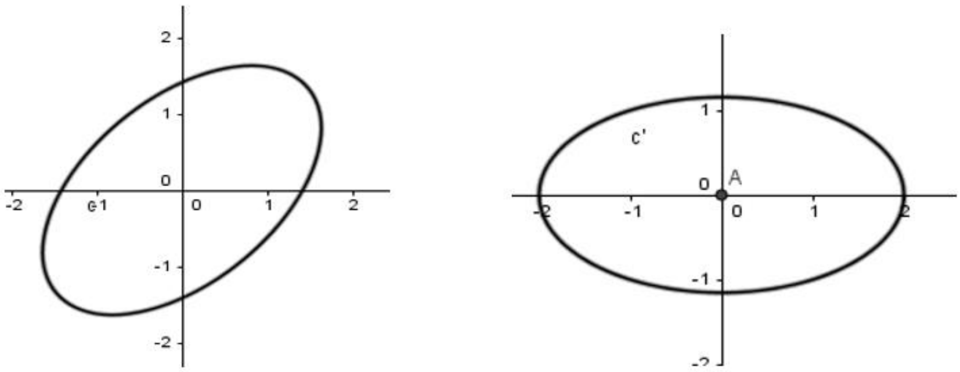


Figure 4. *The second ellipse, the new shape of the first ellipsoid rotated by 45 degrees around the origin.*

According to Ada and Kurtulus (2010, p.91), applying the appropriate coordinate transformation to complicated algebraic equation and reducing to the standard form can determine which curve represents. On the other hand, the new location of the curve can be determined by determining the rotation point, angle and direction of the rotation transformation applied to a set of geometrically defined points. In this context according to Hollebrand (2003), coordinate transformations is important since it provides opportunities for students to consider important mathematical concepts (such as function, symmetry), provides a context in which mathematics can be viewed as an interdependent discipline, and gives opportunity to students for the development of high level reasoning skills.

According to NCTM (2000), coordinate transformations stand out as a concept that assists students to develop especially their spatial reasoning and visualization skills. Curriculum supports to teach rotations to students at all levels (NCTM, 2000;. McNiff, J., Lomax, P., & Whitehead, J., 1996). Furthermore, as examined the literature, it appears that groups of students at all ages have difficulty working with concepts related to rotations (Yanik & Flores, 2009).

In Ada and Kurtulus (2010) study, has been revealed that applying appropriate translation or rotation transformations helped to determine type of curve as algebraically when determining the type of a given curve. It is thought that the equations of curves given in complex form with coordinate transformations by simplifying in standard form will facilitate the determination of the type of curves as algebraically. However, researchers have experienced difficulties in teaching experiences when students used rotation as algebraically or had experienced problems in interpreting.

The purpose of this study is to find a solution to solve the problems encountered in the teaching of rotation of conic sections and how to solve these problems.

2. Methodology

2. 1. Research Model

The researchers have determined the problems that mathematics teacher candidates who are dealing with coordinate transformations have experienced and noted the difficulties they encountered in applying rotations to equations and drawing the graph of the new equation they have acquired. In this study, solutions were developed with the prepared activities to overcome these problems and applied as an action research. The action research has a cyclical characteristic to determine and identify a problem and ensure to gather information, scanning resource, develop and improve problem solving actions (Johnson,2002; McNiff, Lomax, & Whitehead, 1996).

2.2. Participants

A total of 40 third grade elementary mathematics teacher candidates who study at a state university and took Analytic Geometry 1 course in the fall semester of 2016-2017 academic year participated in this study. The prepared lesson plan was applied for three weeks.

2.3. Data Collection

The data were collected from solutions of teacher candidates to open-ended questions during determining the case and solutions to evaluation questions during the implementation process. Moreover, in-class observations were made by researchers throughout the study.

2.4. Data Analysis

In the study, data were obtained through using structured questions and observation technique. The information was interpreted by grouping while data from observations and open-ended queries were analyzed. The data obtained from the structured queries were analyzed descriptively. In descriptive analysis, quotes and examples from students' papers were given in order to clearly reflect the thinking styles of the students, and the data are transferred in original form.

2.5. Research Process

The study process consists of three stages. In the first stage, the concept of rotation transformation in coordinate transformations to teacher candidates was explained theoretically. In the teaching of this subject both algebraic and geometric approaches have been emphasized. In the algebraic approach, the rotation transformation relations and the change in equations of the curves were given. The rotation transformation relation is defined (*) below.

$$R(\theta):\begin{cases} x &= x'\cos\theta + y'\sin\theta \\ y &= x'\sin\theta - y'\cos\theta \end{cases} \ldots\ldots(*)$$

Two approaches were considered as geometrically (Kaya, 2012). The first is to rotate the coordinate axes around the origin. In this case, the image under the rotation with the set of points given is the same (Figure 5).

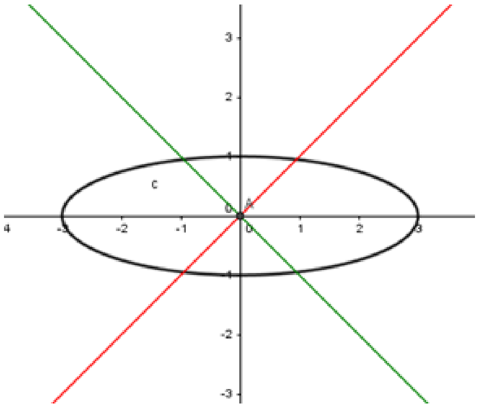


Figure 5. *Coordinate axes rotated around the origin with a certain angle*

The second is to rotate the set of points around the origin. In this case, the set of points and image is not same (Figure 6).

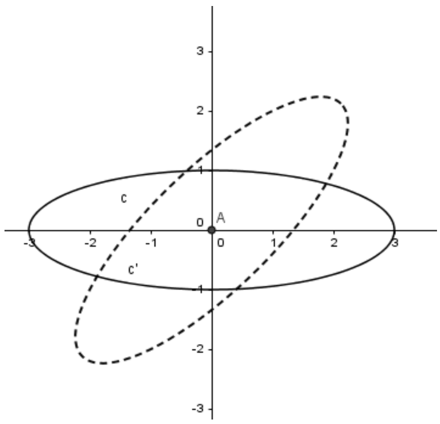


Figure 6. *The ellipse rotated around the origin by a certain angle*

In the first approach, the coordinate axes only were rotated and the location of the curve did not change. In this study, the second approach was considered since the conic sections aimed to observe coordinate transformations and change of their locations.

The second stage is determining the case. At this stage, a test consisting of open-ended questions has been implemented to determine whether the teacher candidates who probed these approaches of rotation transformation understand this subject to what extent. Accordingly, some of the teacher candidates have incorrectly plotted the graphs of the curves while they did algebraic operations correctly in the solution of equation solutions of conic sections.

Geogebra software which is one of the dynamic geometry software besides algebraic operations in the solution of the equations of the curves required to determine their types in the application stage which is the third stage has been applied in order to solve these problems which the teacher candidates have experienced geometrically. Open-ended questions were asked to the teacher candidates by researchers in order to observe whether these applications were sufficient or not.

3. Findings

In this study, the research process described in the previous chapters was followed and the class was carried out in accordance with the correction of mistakes in coordinate transformations on the plane. Coding was used for participants. Findings from determining the case and application phases of action plan were given below in order.

Determining the Case and Feedback (Second stage): In this stage, teacher candidates were asked to define the type and graph of a curve whose algebraic equation is given in an open ended question based test. A short period of time was provided for teacher candidates to solve the question. During this period in the first question, some teacher candidates tried to round the given equation to a completing the square while some did nothing. In figure 7, it can be seen that S11 coded teacher candidate tried to round the equation to completing the square but could not come up with the standard format.

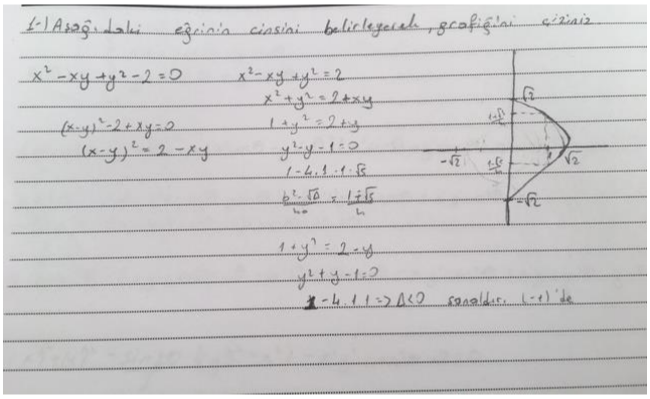


Figure 7. *Solution of the S11 coded teacher candidate*

Teacher candidate inserted the equation of curve in geogebra and obtained the graph. Thereby the type was determined. "*What needs to be done to bring this equation to standard format?*" was answered as transformation application. To this equation, teacher candidates stood by the solution of coordinate transformation. Based on that, some candidates applied the transformation of parallel and vertical translational transformation while others applied rotation transformation. Solution of the S30 coded candidate, whom applied the translation transformation, is illustrated in figure 8. When the solution is examined, the formula of in parallel and vertical translation transformation ($T_{(a,b)}: \begin{cases} x = x' + a \\ y = y' + b \end{cases}$) is applied. Therefore, by finding the values of a and b, the candidate tried to get rid of x and y terms. However, since the transformation was not correct, equation did not come in to standard format.

In this case, an explanation was made by researchers saying that the transformation should be picked based on the terms of equation of curve and in order to get rid of terms including "xy", rotation transformation would be more appropriate. Thus, those with wrong solutions exercised the transformation by rotation and brought the equation to standard format. Figure 9 shows the correct solution of S11 coded teacher candidate.

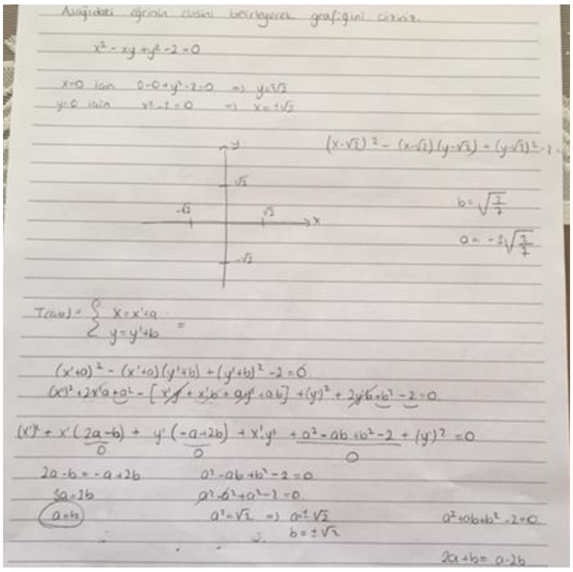


Figure 8. *Solution of the S30 coded teacher candidate*

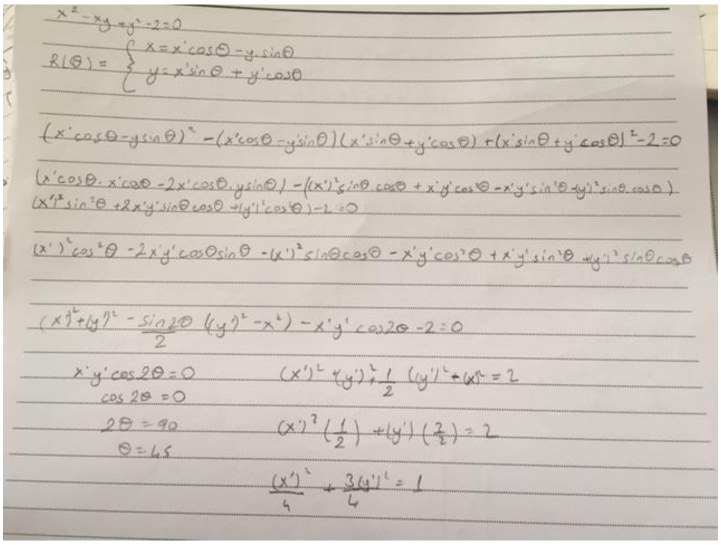


Figure 9. *Correct solution of S11 coded candidate*

Even though some teacher candidates applied the appropriate transformation on equation, they could not come up with the standart format because of calculation errors. Figure 10 shows the solution of S22 coded candidate whom applied the transformation by rotation but attained the wrong equation and therefore drew a mistaken graph because of calculation errors.

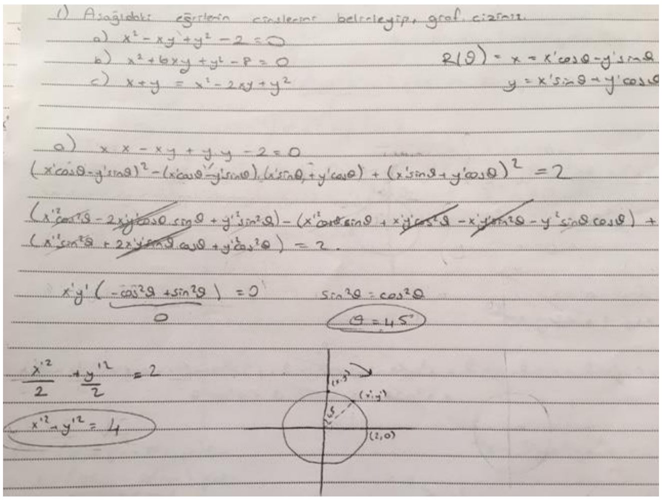


Figure 10. *Solution of the S22 coded candidate*

When asked to draw the image and the body of the same curve in the same coordinate system, some teacher candidates were not able to insert the standard graph of the curve in correct location on the coordinate system. Some students, on the other hand, adopted the first approach of transformation by rotation which involves the rotation of coordinate axes with curves' location staying in the same position. However, in this study, second approach of rotation was based in the evaluation. For example, as shown in figure 11, S14 coded teacher candidate determined the wrong graph of curve since the focuses had to be on coordinate axes. Figure 12 shows the S15 coded teacher candidate's solution that uses the first approach of transformation by rotation.

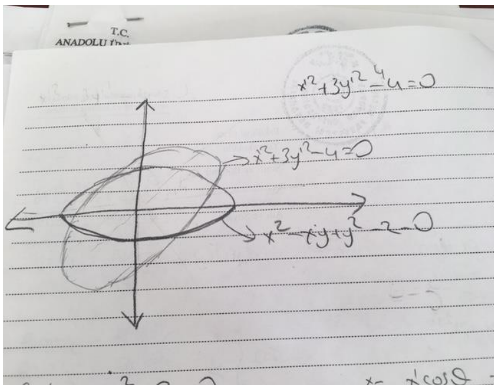


Figure 11. *Solution of the S14 coded candidate*

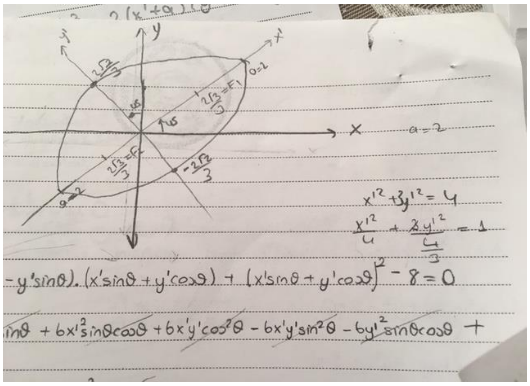


Figure 12. *Solution of the S15 coded candidate*

Application (Third stage): In this step, firstly in order for those who were not able to draw the graph in the previous step correctly to notice the right drawing, using the Geogebra software of Dynamic Geometry, the image of curve while the graph is being rotated around its starting point with the angle of 45 degrees has been drawn. After that, another test of open ended question similar to first one was applied to teacher candidates.

Most of the teacher candidates successfully brought the equation to standart form by applying the appropriate transformation of rotation to the curve. However, when it came to the drawing of graph, troubles were confronted one more time. To overcome these problems, every participant of the study entered the equation of the curve in Geogebra software and observed the graph of curve and the angle in which the curve's image is being obtained. Drawings of teacher candidates in application step on Geogebra software are presented in Figures 13, 14 and 15.

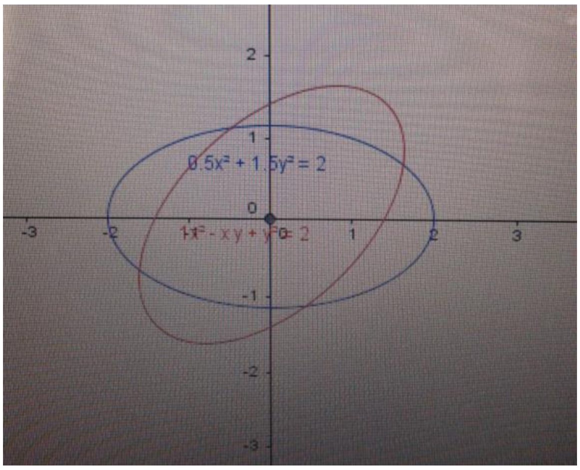


Figure 13. *Graphy and standart form graphy of elipse*

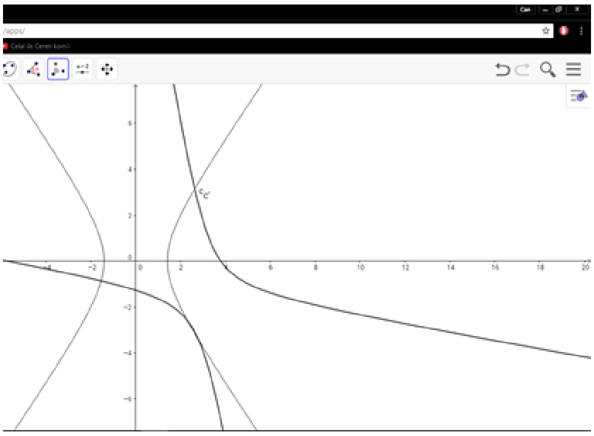


Figure 14. *Graphy and standart form graphy of hyperbola*

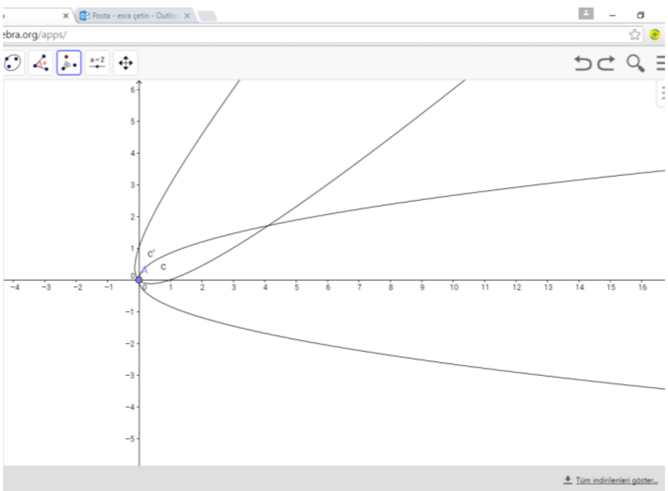


Figure 15. *Graphy and standart form graphy of parabola*

A final open ended questions test was applied to teacher candidates for the purpose of understanding how helpful the solution methods had been in overcoming the troubles faced in determining the case step. Acquired data were analyzed descriptively. Themes were created based on the solutions of teacher candidates. These themes are stated in Table 1.

Table 1. *Distribution table of 40 teacher candidates according to the answers given to the questions*

A single curve in both coordinate systems	**Correct answer**	**Right solution but errors in calculation**	**Incorrect answer**	**Operational solution exists but graph drawing is incorrect**
S30, S38, S37, S36, S29, S9, S6, S4	S1, S23, S24, S25, S26, S27, S28, S5, S7, S8, S10, S11, S12, S13, S14, S15, S16, S18, S19, S22, S21, S20, S17	S2, S3	S39, S40	S31,S32, S33, S34, S35

Examining the Table 1, 23 out of 40 students answered the question correctly. They applied the suitable transformation to the equation and the curve was concluded in standard format. Both the graph of curve and the image were drawn.

Figure 16 shows the solution of S8 coded teacher candidate who answered the question correctly. This teacher candidate applied the transformation of rotation in the first step but because of calculation errors he/she attained an incorrect equation and drew the graph incorrectly. As a result of applications took place until the process of open ended question test, he/she was seen to overcome the errors as seen in figure 16. S8 coded teacher candidate applied the appropriate transformation to equation of curve and reached the standard form of equation. This teacher candidate also managed to draw the image and graph of the equation in the same coordinate system correctly.

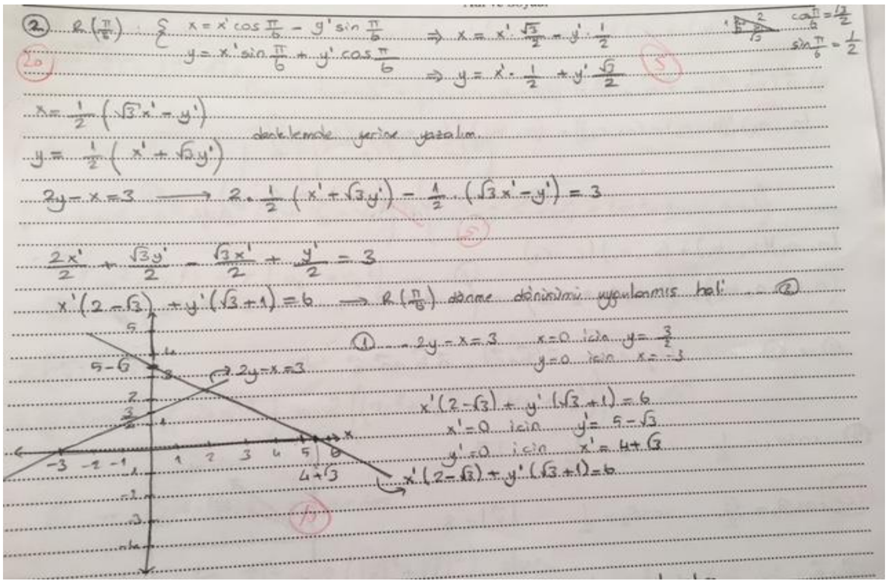


Figure 17. *Solution of S6 coded teacher candidate*

8 out of 40 candidates in this study partially answered the question correctly. They applied the right transformation of rotation appropriate to the equation of curve and reached the standard format but applied the first approach of rotation transformation, therefore their answers were not regarded as totally correct. As an example to this case, figure 17 shows the solution of S6 coded student's solution.

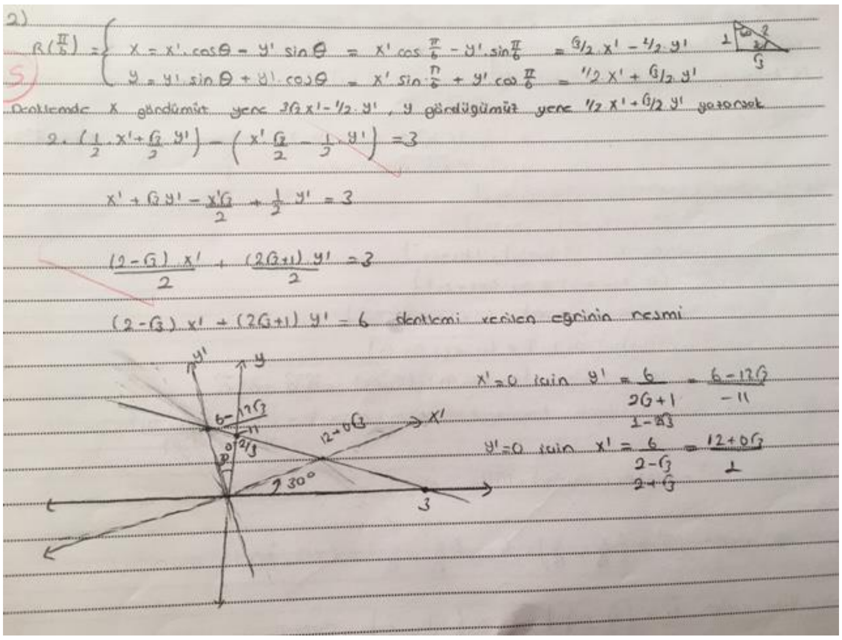


Figure 17. *Solution of S6 coded teacher candidate*

S6 coded teacher candidate rotated the coordinate axes around origin with 30 degrees instead of rotating the curve around origin with 30 degree angle. This solution is not incorrect. However, in our study, the goal is for teacher candidates to recognize how the position of curve changes under rotation transformation. Thus, the solution of teacher candidate does not include the second approach.

5 participant teacher candidate applied the appropriate rotation transformation to given equation and reached the standart format of the equation. However in geometric design, it was noticed that the curve was drawn incorrectly without knowing what the curve implies. As examples of this case, figure 18 and figure 19 shows the solutions of S35 and S34 coded teacher candidates' solutions respectively.

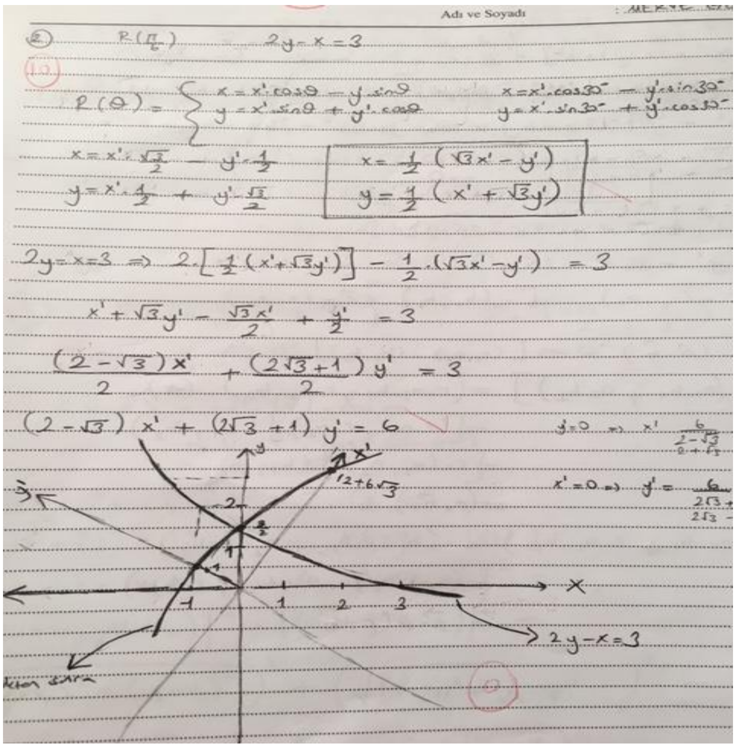


Figure 18. *Solution of the S35 coded teacher candidate*

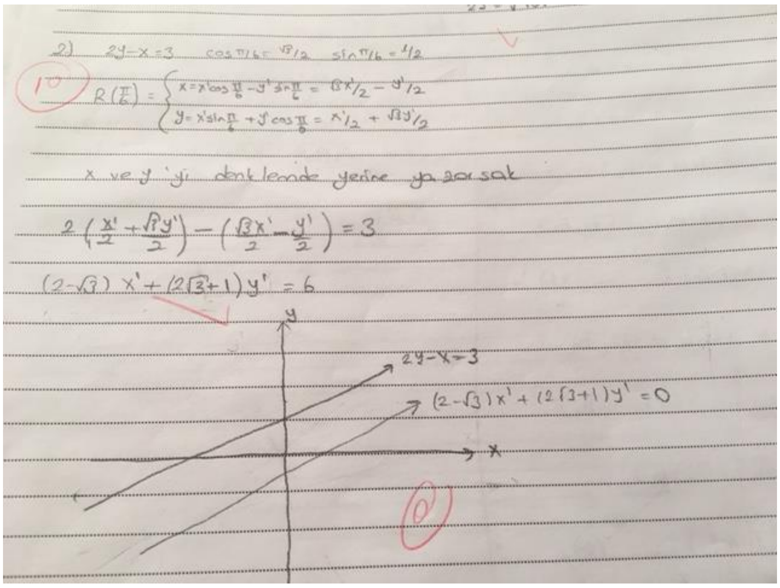


Figure 19. *Solution of the S34 coded teacher candidate*

S35 coded teacher candidate' solution was flawless until the drawing of the graph but instead of drawing the graph of line y=2x-3, the teacher candidate drew a curve of parabola as seen in fig.18. It is obvious that this candidate does not realize the purpose of his/her algebraic operations. S34 coded teacher candidate also conducted the algebraic operations rotely but mistook the geometric demonstration of rotation transformation with translation transformation as seen in figure 19.

2 of the participant teacher candidates also made calculation errors although the course of their solutions were correct. These errors led them to find standard format of equation incorrectly. They drew the right graphs of curve for their equations. As an example of this situation, figure 20 shows the solution of S3 coded teacher candidate's solution.

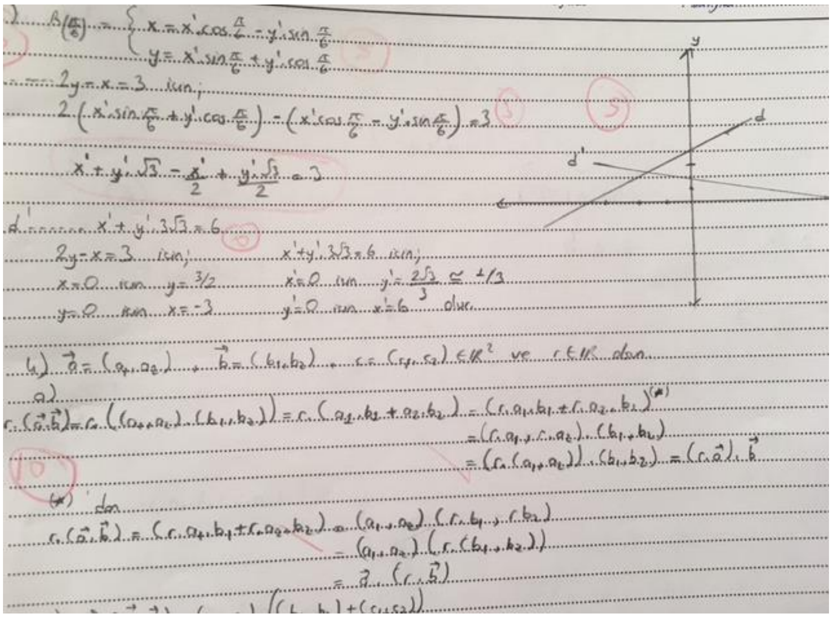


Figure 20. *Solution of S3 coded teacher candidate*

2 of the 40 participant teacher candidates answered the question incorrectly. S39 coded teacher candidate illustrated in figure 21 does not know about the rotation transformation formulas. The candidate was observed to take the image of a point on the curve to complete the transformation.

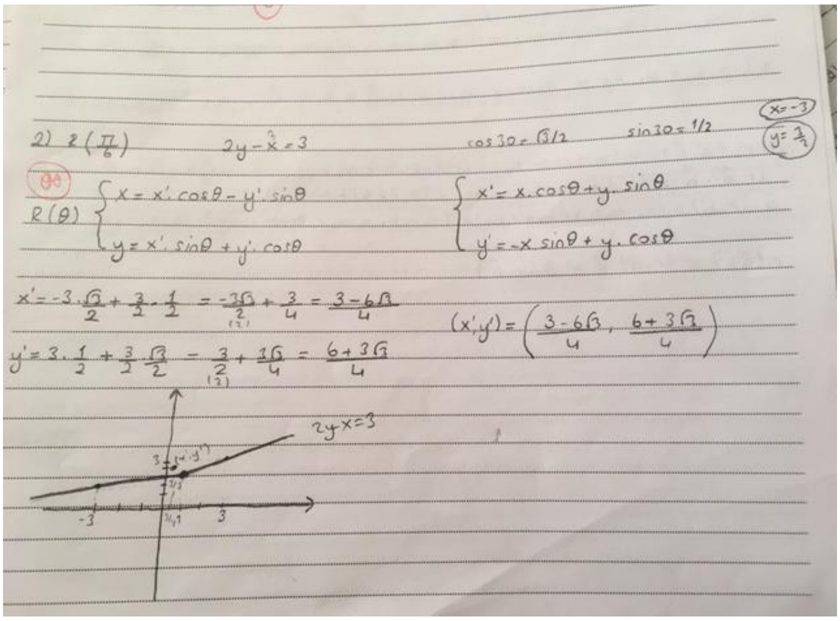


Figure 21. *Solution of the S39coded teacher candidate*

4. Conclusion

Analyzing the responses of teacher candidates to open ended questions, at first they were noticed to have difficulties in determining the type of given curve. Therefore, an action plan was developed. In the early stages of action plane, teacher candidates were not paying attention to the fact of which transformation would help get rid of which terms in order to bring the equation to the standard format. Thus, they had a hard time picking the right transformation. In this concept, with the activities that the researcher carried out, they witnessed the changes in the equation of curve with transformation by rotation formulas. Even though some teacher candidates applied the appropriate transformation of rotation, they still could not come up with the correct graph because of calculation errors. To overcome these errors, utilizing Geogebra dynamic geometry software in the application step, line graph of the curve was drawn. So, by rotating the curve around origin with a specific angle, standard graph was captured. When asked to draw the image and the body of the same curve in the same coordinate system, some teacher candidates were not able to locate the standard graph of the curve in its position on coordinate system correctly. Some, on the other hand, adopted the first approach of rotation transformation which involves the rotation of coordinate axis while the curve being kept in the same position. However, since the approach in the study aims to reveal the changes in positions and conical sections, these candidates' solutions were not counted as true. The standard format equation of the curve coming from rotation transformation and with the drawings of the curve in Geogebra, the second approach was utilized. In the second step, to investigate how effective the activities had been, examining the last open ended test, most of the participant teacher candidates were seen understanding both algebraic and geometric approach of the rotation transformation. Still, when they were asked to draw the curve and standard graph, some teacher candidates rotated coordinate axes and kept the curve in the same location. Since there are two approaches to the transformation by rotation, this situation does not prove that those teacher candidates did not comprehend the transformation by rotation. As seen from Table 1, just a few teacher candidates had calculation errors and thus having the wrong equation and drawing an incorrect graph and not resulting in a solution set. The repetition of mistakes in graphic drawings shows the lack of knowledge of students in this subject. Teacher candidates may register the courses including these topics to handle their lack of knowledge.

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