Acta Didactica Napocensia
Volume 14, Number 2, 2021 - DOI: 10.24193/adn.14.2.12

Development of Multiple Representation Translating Measurement Tool and Examination of 9th Grade Students' Multiple Representation Translate Skills in Algebra

Sevcan MERCAN ERDOĞAN, Hatice ÇETİN, Kamil ARI

Abstract: In this study, it was aimed to examine 9th grade students' multiple representation translate skills in algebra learning area and the relationship between these representations translate skills. The study employs survey design. Selected through purposive sampling method, 637 ninth grade students in a socio-economically medium level province in Turkey participated to the study. Within the scope of the study, Multiple Representation Translating Measurement Tool (MRTMT) including four factors (verbal- graph- algebraic- table) was developed and data were collected through this instrument. The validity and reliability of the scale was tested with Rasch analysis and confirmatory factor analysis (CFA). Cronbach Alpha value was calculated as .88. The findings show that students' skills of translating between representations level is low. In addition, students were most successful in expressing the situation with other representations when the table representation was given, but had difficulty in translating it to other representations when given an algebraic representation. Furthermore, it is found out that there is a significant relationship between students' translating skills to verbal- graph- algebraic- table representations. Accordingly, it can be suggested that each representation should be used in an interrelated and holistic way in algebra teaching.

Key words: Algebra, measurement tool, translate among multiple representations, 9th grade.

1. Introduction

Mathematics is an abstract discipline and has its own terminology. Algebra is a learning area where we use this terminology most in mathematics. In the most general sense, algebra is the language of mathematics and this language consists of meanings and operations (Baki, 2018). In addition, algebra has been recognized as a critical turning point in students' mathematics learning (Wang, 2015). On the other hand, it is important to teach algebra with understandable representations since it is considered incomprehensible and abstract. Accordingly, because algebra is perceived as abstract by students, it is stated that there is a need to use multiple representations effectively and to associate these representations in algebra teaching (Sarpkaya Aktaş, 2019). As a matter of fact, the conceptual learning desired in mathematics teaching is possible by understanding the problem situation with various representations and translating the knowledge to different representations (Bossé, Adu-Gyamfi & Chandler, 2014; Keller & Hirsch, 1998; Ural, 2012). Multiple representations are used extensively in mathematics (Gagatsis & Shiakalli, 2004). The use of multiple representations in many fields of mathematics education and the investigation of translate skills give important clues about the conceptual and meaningful learning of the subject studied.

In the literature, we often come across studies investigating translate skills between multiple representations in algebra teaching. A review of literature suggests that the studies mostly focus on elementary education (Akkuş & Çakıroğlu, 2006; Demir & Cansız Aktaş, 2019; Deniz, 2016; Gurbuz & Sahin, 2015; Mourad, 2005; Sarıhan Musan, 2012; Sert, 2007) and higher education levels (Andrá, Lindström, Arzarello, Holmqvist, Robutti & Sabena, 2015; Bal, 2015; De Bock, Van Dooren & Verschaffel, 2015; Galbraith & Heines, 2000; Ipek & Okumus, 2012; Kardes, Aydin & Delice, 2012;

Received April 2021.

Cite as: Mercan Erdogan, S., Çetin, H. & Ari, K. (2021). Development of Multiple Representation Translating Measurement Tool and Examination of 9th Grade Students' Multiple Represenations Translate Skills in Algebra. *Acta Didactica Napocensia*, 14(2), 160-180, https://doi.org/10.24193/adn.14.2.12

Presmeg & Nenduradu, 2005). In these studies, multiple representation translates were generally identified by adopting qualitative methods (Deniz, 2016; Fonger, Davis, Rohwer & Lou, 2018; Gurbuz & Sahin, 2015; Huntley & Davis, 2008; Ipek & Okumus, 2012; Presmeg & Nenduradu, 2005; Rahmawati, Purwanto, Subanji, Hidayanto & Anwar, 2017). Multiple representation translate tests developed in mathematical contexts in the field of algebra (Bal, 2015; Demir & Cansız Aktaş, 2019; Kardeş et al., 2012; Ozhan-Turan, 2011) are available in the literature. However, a valid and reliable instrument that verifies the multiple representation components (verbal-graph-algebraic-table) at secondary education level is not available in the literature, to the researcher's best knowledge. On the other hand, in secondary education curricula, there are many learning outcome guidelines that encourage the use of instructional technologies with which we can easily integrate multiple representations in algebra teaching (MoNE, 2018; NCTM, 2000) because secondary level students are ready for this. It is believed that it is sufficient for a high school student to show skill only in algebraic operations. However, it is important to get them provide not only operational mechanics but also translates between representations for conceptual learning (Duval, 2006). Many researchers draw attention to the importance of representations in mathematical understanding and associate the understanding of mathematical knowledge with representations (Adiguzel & Akpinar, 2004; Goldin, 2003; Janvier, 1987; Parrot & Leong,2018; Schultz & Waters, 2000). This study focuses on translate skills between multiple representations, which are stated to have an important role in critical mathematical skills such as conceptual learning, problem solving, and relating (Bossé et al., 2014; Keller & Hirsch, 1998; Ural, 2012). Since students experience challenges in algebra which is thought to be abstract (Lew, 2004; van Ameron, 2002; Witzel, Mercer, & Miller, 2003), the current research has been carried out on the examination of multiple representations and their translate, which is a method to overcome these difficulties.

In this study, the process of the original development of the multiple representation translate test for 9th grade students and examination of 9th grade students' multiple representation translate skills in the field of algebra learning through the quantitative research method- unlike previous research- are reported. Multiple Representation Translating Measurement Tool was originally developed in this study based on verbal- graph- algebraic- table multiple representation components (Bossé, Adu-Gyamfi & Cheetham, 2011a; Heinze, Star & Verschaffel, 2009; Waisman, Leikin, Shaul & Leikin, 2014; Wilkie, 2016) in line with equations and disequilibrium sub-learning areas in secondary education 9th grade curriculum (MoNE, 2018). In addition, it was sought to answer the following research questions: "What is 9th grade students' level of multiple representation translate skills in algebra learning area?" and "Are there relationships among 9th grade students' verbal- graph- algebraic- table multiple representation skills in algebra learning area?"

2. Theoretical Framework

2. 1. Representation, Multiple Representation, Translate

Representation is a structure that replaces another one, such as a word for an object, a sentence for a situation, a scheme for the arrangement of things, a picture for a scene (McKendree, Small, Stenning & Conlon, 2002). Visual representations can differ in various design dimensions: they can be concrete or abstract; they can be physical (i.e. visual features are concrete) or virtual (i.e. visual features are presented on a digital display); they can be static or animated (i.e. visual properties change during the learning experience); and they can be interactive (Rau, 2017). There are different classifications of the concept of representation in the literature. One of these classifications is the classification made by Lesh, Post & Behr (1987) consisting of pictures and diagrams, manipulatives, verbal symbols, written symbols and real life situations components. Kaput (1998) stated that such a classification is cognitive in essence and does not take into account other perspectives related to thinking and learning (cognitive, behavioral, etc.). Lesh, Post & Behr (1987) expresses representation as external practices of students' inner understanding. While Wileman (1980) claims that there is no difference between internal (mental) representations and external representations, and internal representation is the image of the external representation in the mind (cited in: Bayik, 2010), some researchers believe that students' mental development is directly related to their ability to work (Pape & Tchoshanov, 2001). Students can be more actively involved in learning to construct and interpret representations by discussing the properties of representations, including their advantages and limitations. (Greeno & Hall, 1997, p. 362).

The concept of multiple representations in mathematics education is emphasized by many researchers (Aviles-Garay, 2001; Bieda & Nathan, 2009; Dundar, 2015; Even, 1998; Gagatsis & Shiakalli, 2004; Hitt, 2002; Kaya, 2015). Researchers stated that learning with multiple representations is necessary for a conceptual understanding of mathematics (Adu-Gyamfi et al., 2015; Bossé et al., 2014; Dreher, Kuntze & Lerman, 2015; Keller & Hirsch, 1998). The reason why multiple representations are emphasized by researchers is that a single representation cannot express a mathematical item clearly and saliently. At this point, it was suggested that in order to learn a concept, it should be constructed by diversifying the concept with its external representations as much as possible, and that more opportunities should be given for translate between representations (Hitt, 2002).

Translate is defined as the transition from one representation style to another (Janvier, 1987) and "movement required by an interpretation into a different modality of representation" (Roth & Bowen, 2001, p. 161). What is translated is not the representation itself, but the ideas or structures expressed through representations (Adu-Gyamfi, Stiff & Bossé, 2012). The term translate is the successful transformation of the information and relationships expressed by a mathematical representation into a target representation. The term translates used in this study refers to matching the structures of a mathematical representation with another representation. An example of this process is to take a statement presented in tabular form and express it graphically while maintaining its rule or relationship. Thus, the translate process includes two forms of representation, an initial (input) representation and a target (output) representation (Adu-Gyamfi et al., 2012). The translate will be successful if the basic elements or structures expressed in the initial representation are successfully expressed through the structures in the target representation.

2.2. Multiple Representation and Algebra

Students perceive algebra as a very difficult lesson (Egodawatte, 2011; Kalaivani & Tarmizi, 2014). In particular, teaching algebra as a subject dealing only with abstract symbols supports students' belief that mathematics does not make sense (Lew, 2004). The concept of variable which involves symbols is considered one of the most difficult and important concepts in algebra. Therefore, students have difficulty in learning this subject conceptually. If abstract concepts are taught to students in a concrete way, this difficulty can be eliminated or reduced (Kalkan, 2014). At this point, teachers can encourage students to think by choosing problems involving meaningful real-life situations and a teaching method that encourages students to think in different ways.

The purpose of supporting translates between multiple representations is to enable students to build their representations onto each other’s to develop strong mathematical ideas, instead of making mathematical discussions with different methods using separate representations to solve a specific problem (Stein, Engle, Smith & Hughes, 2008). Translating multiple representations in mathematics teaching gives the opportunity to conceptualize, express and observe mathematical concepts in different ways. Hitt (1998) supported this view and stated that the main purpose of teaching mathematics is to enable students to pass from one representation type to another without falling into contradictions. Many teaching approaches have been proposed by the mathematics education community to address this current problem. The key role of learning mathematics, emphasized by many researchers in the field of mathematics education, is also expressed in the current national standards of many countries (e.g. NCTM, 2000; Education Minister Confererence- Kultus Minister Konferenz [KMK], 2003). As a component of mathematical competency, KMK expressed using mathematical representations as:

- Implementing, interpreting and distinguishing different representations for mathematical objects and situations,
- Recognizing the connections between representations and
- Choosing different representations and changing between them depending on the conditions and purpose (KMK, 2003, p. 8).

In studies on algebra, we usually see the multiple representation components as verbal-graph-algebraic-table representations (Bossé et al., 2011a; Heinze et al., 2009; Waisman et al., 2014; Wilkie 2016). For this reason, in this study, the research was conducted with a measurement tool which was verified by referencing these four representations.

The purpose of this study is to examine 9th grade students' multiple representation translate skills in algebra learning area and the relationship between these representations translate skills by the original developed tool (MRTMT). The researh questions were determined as;

- What is 9th grade students' level of multiple representation translating skills in algebra learning area?
- Are there relationships among 9th grade students' verbal- graph- algebraic- table multiple representation skills in algebra learning area?

3. Method

3.1. Research Design

In this study, 9th grade students translate skills between representations in algebra were examined through survey model within the scope of quantitative research methods. Survey design research studies are mostly carried out with descriptive purposes and put forth the picture of a case at a specific time (Robson, 2017). This study reveals 9th grade students' level of translate skills between multiple representations in algebra without experimental intervention to conditions, objects and individuals.

3.2. Participants

Selected through purposive sampling method, 637 ninth grade students studying at different types of high school in a socio-economically medium level province in Turkey participated to the study. Accordingly, the criteria were studying at ninth grade and different types of high schools. Participants' distribution in terms of gender is presented in Table 1.

Table 1. *Participants' distribution in terms of gender*

Gender	Frequency (f)	Percentage (%)
Female	332	52.1
Male	305	47.9
Total	637	100.0

According toTable 1. 332 (52.1%) female and 305 (47.9%) male students participated in the study. In addition, the school distribution of the students participating in the study is given in Table 2.

Table 2. *Participants' distribution in terms of schools 2.*

School	Frequency (f)	Percentage (%)
School 1	84	13.2
School 2	110	17.2
School 3	55	8.6
School 4	92	14.5
School 5	42	6.6
School 6	98	15.4
School 7	76	12
School 8	41	6.4
School 9	39	6.1
Total	637	100

As is given in Table 2, of the participants in the study, 84 (13.2%) were from the first high school, 110 (17.2%) were from the second high school, 55 (8.6%) were from the third high school, 92 (14.5%) were from the fourth high school, 42 (6.6%) were from the fifth high school, 98 (15.4%) were from the first sixth school, 76 (12%) were from the seventh high school, 41 (6.4%) were from the eighth high school, 39 (6.1%) were from the ninth high school. Considering the research questions, school types were not expressed here as students' schools are not differentiated in the analysis**.**

3.3. Data Collection

The data were collected from participants who voluntarily accepted to participate to the study. The data collection was held in a mathematics lesson in the spring semester of 2018-2019. It took about 45 minutes. All ethic rules were followed and taken ethic permission from the authorities.

3.3.1. Development of the instrument

Due to the lack of a valid and reliable Multi-representation Translating Measurement Tool at the 9th grade level in the literature, it was decided to develop the MRTMT originally to search for answers to research questions. The measurement tool was developed in six phases. In the first phase, the researcher and field specialists agreed on develop content for the learning outcome "Practices are performed on algebraic, graphical and numerical representations of the relationships in verbal expressions representing real life situations.", which is present in numbers and algebra learning area of MoNE secondary education mathematics curriculum (MoNE, 2018, p.21). To this end, a group of specialists was formed. The group consisted of a faculty member who has a PhD in mathematics teaching, a faculty member who has a PhD in algebra, a Turkish language specialist and a high school mathematics teacher. In the second phase, the group came together and identified the outcomes needed in multiple representations translates. Accordingly, they agreed on 16 questions. In the third phase, a Turkish language specialist controlled the questions. In the fourth phase, the questions were presented to prospective elementary school mathematics teachers. Face validity of the questions were ensured by controlling their clarity and intelligibility. In the fifth phase, the researchers came together and revised the questions based on the feedback from prospective teachers and Turkish language specialist. After the discussion, 4 questions were discarded since they measured the same aspects. In the sixth phase, the test including 12 questions was presented to two high school mathematics teachers. Following the teachers' examination, multiple representations translating measurement tool was formed. In order to finalize the draft scale after statistical procedures, a pilot study was carried out. First, the measurement tool was implemented with 97 high school students and the Cronbach alpha value was calculated as .88.

MRTMT including 12 open ended questions consists of four parts. In the first part, students are provided with a verbal problem and the students are asked to express this problem in algebraic, table and graph forms. In the second part, students are asked to express a problem given in the form of a graph in verbal, algebraic and tabular forms. In the third part, students are asked to express the problem given in algebraic form in verbal, table and graph forms, and in the last part, they are asked to express the problem given in the table representation in verbal, algebraic and graph forms (see Annex).

3.3.2. Validity and Reliability of the Measurement Tool

The Rasch Measurement model was used to determine the item difficulty and reliability values, item discrimination values of the items and validity and reliability of MRTMT, developed within the scope of the research (Rasch, 1980). The Rasch model examines whether the data fit to the model (Linacre, 2008), the difficulty level of the items, and the linear relationship between item difficulty and items (Dervent, Devrilmez, İnce & Ward, 2018). Rasch analysis was analyzed with Winstep 3.72.4 and its findings are shown in Table 3.

Table 3. *Item difficulty, fit indices and score-measurement correlations*

Item No	Total Score	Item Diff.	Std. Error	Infit		Outfit		Score-Measurement Correlation
				MNSQ	ZSTD	MNSQ	ZSTD	
4	44	2.67	0.84	1.44	1.4	*1.79*	1.6	.23
6	51	2.34	0.34	1.36	1.9	1.42	1.8	.35
12	70	1.88	0.16	1.36	1.4	1.24	1.1	.42
11	56	1.06	0.18	1.03	.3	1.34	1.4	.38
10	74	1.61	0.16	1.15	1.1	1.33	1.4	.27
3	72	0.69	0.26	1.01	.1	.82	-.6	.50
5	51	0.75	0.16	1.01	.1	.94	-.2	.59
7	97	0.57	0.13	.97	-.1	.94	-.3	.59
2	66	-0.62	0.23	.95	-.3	.72	-1.1	.63
1	59	-0.64	0.19	.61	-2.0	.84	-.6	.51
8	65	-1.83	0.12	.78	-1.7	.70	-1.7	.61
9	63	-2.33	0.11	.69	-1.9	.63	-2.0	.65

In Table 3, item numbers are in the first column; total scores of correct answers are in the second column and item difficulty index is in the third column. The fourth column includes standard error regarding difficulty of the questions. The fifth and sixth columns present the internal and external fit indices. Internal fit is sensitive to expected responses, while external fit is sensitive to unexpected responses (Linacre, 2008). The fit indices section provides the content validity of the measurement tool by showing the consistency between the questions. In the chart of fit index, the questions from top to bottom are listed from hard to easy. Mean square values (MNSQ) show how the obtained responses form a pattern of test items with regard to model fit. MNSQ is expected to be between 0.5 and 1.5 (Linacre, 2008). Standardized distribution values (ZSTD) indicate the meaningful compatibility levels of test items with each other. According to Bond and Fox (2015), this value should be between +2 and -2. The last column gives the score-measurement correlation values. The positive values indicate that the correlation is in the expected direction.

According to Table 3, it is seen that 11 items of the 12-question test provide the expected MNSQ and ZSTD values. Only in the external fit of test item number 4, the MNSQ value is slightly above the expected value. Internal fit is more sensitive than external fit and is more important for the validity and reliability of test items (Bond & Fox, 2015). Therefore, it seems that the 4th test item does not need to be removed from the test. In addition, all of the score-measurement correlation values are positive.

Table 4. *Difficulty and reliability values of the items and score-measurement correlations*

	Total Score	**No of Respondents**	**Item Diffuculty**	**Standard Error**	**Infit**		**Outfit**	
					MNSQ	**ZSTD**	**MNSQ**	**ZSTD**
Mean	64.0	120.0	1.91	0.24	1.03	-0,13	1.06	0.04
Population SD	13.89	.0	0.62	0.21	0.26	1.60	0.36	1.39
Sample SD	13.30	.0	0.56	0.20	0.25	1.53	0.34	1.33
Maximum	44.0	120.0	2.67	0.84	1.44	1.9	1.79	1.8
Minimum	97.0	120.0	-2.33	0.11	.61	-3.1	.63	-2.0

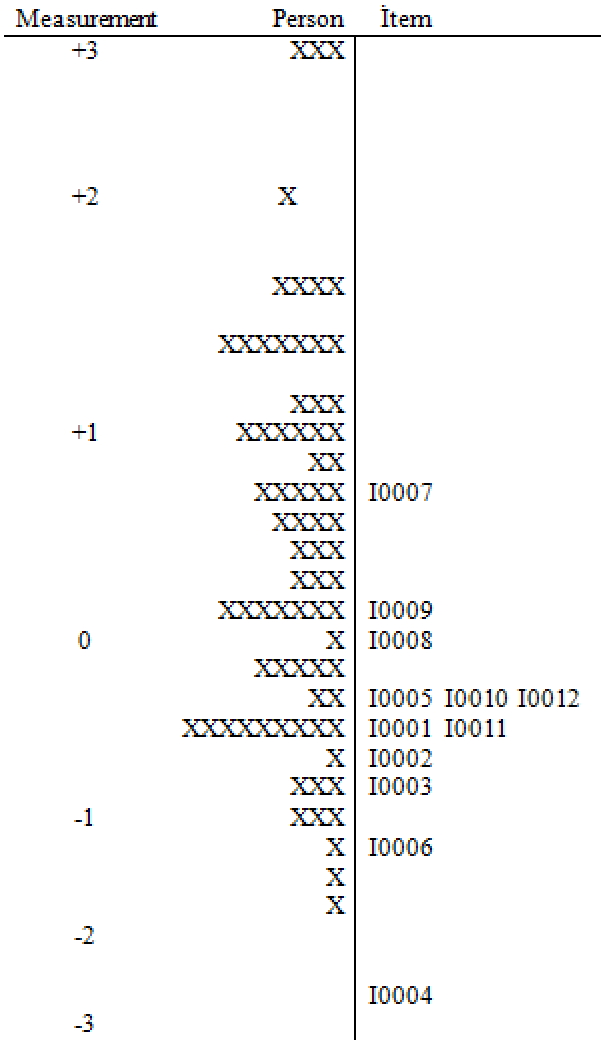
Real RMSE	1.35	Correct SD	3.78	Discrimination	2.81	Item Reliability	.90
Model RMSE	1.25	Correct SD	3.81	Discrimination	3.04	Item Reliability	.91
Standard Error of Item Mean	1.21						

Table 4 shows the difficulty and reliability values of the questions and their score-measurement correlations. Boone, Staver & Yale (2014) propose that the real RMSE values are more consistent than the model RMSE values in research and they should be used.

The representativeness of the items is determined by the item discrimination value. This value is expected to be 2.0 and above (Baghaei & Amrahi, 2011). According to the table, the item discrimination value was determined as 2.81. This value shows that the discrimination level of the items in the test is at the expected level. The item reliability value in Table 4 is accepted as equivalent

to the Cronbach alpha value. The reliability of the items is at the expected level with .90. This value indicates that the internal consistency level of the test is high.

Figure 1. *Person-item Map (Wright Map)*



Person-item map (Wright Map) presented in Figure 1 shows item and person measurements that are in the same direction (Linacre, 2008). The map shows the distribution of items according to their difficulty and the level of matching of these questions with persons. The last column lists the difficulty levels of the items. In this column, item difficulty increases as you go from bottom to top. Accordingly, it can be said that item number seven is the most difficult question and item number four is the easiest question. On the other hand, confirmatory factor analysis (CFA) was performed with Lisrel 8.80 program to check whether the determined factors work in the measurement tool. For this, normality assumptions were provided first and it was decided to analyze 637 data in total. The KMO value was found to be .89, and the Cronbach Alpha value was .88.

CFA enables the researcher to test whether the data is consistent with the previously determined factor structure (Meydan & Şeşen, 2016). As the developed MRTMT was defined as including four factors which are verbal-graph-algebraic-table multiple representation dimensions as given in the literature (Bossé et al., 2011a; Heinze et al., 2009; Waisman et al., 2014; Wilkie 2016), confirmatory analysis was performed. CFA diagram is provided in Figure 2.

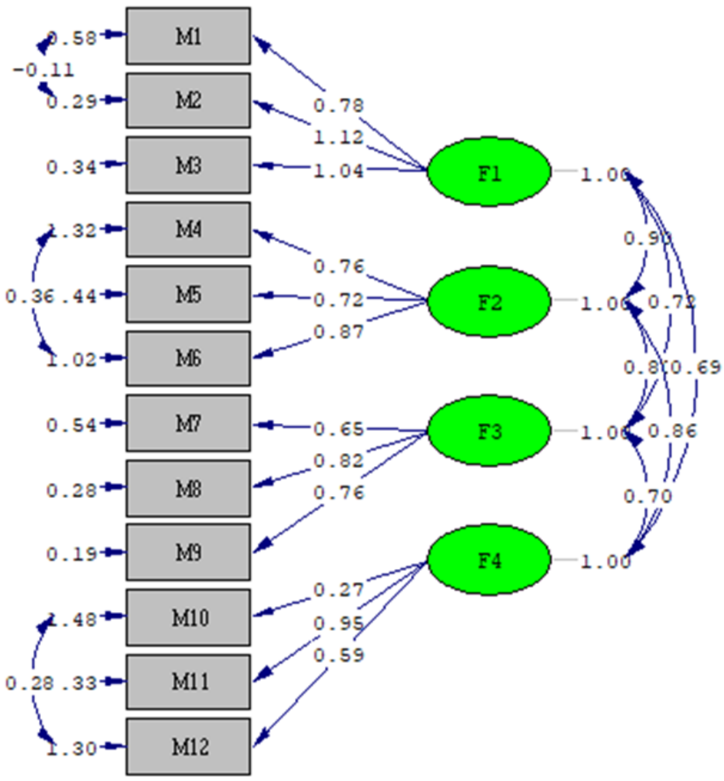


Figure 2. *CFA Diagram*

According to the figure above, M1, M2, M3,…,M12 represent the items of the measurement tool (see Annex). F1 is the skill of translating from verbal representation to other representations, F2 is the skill of translating from graph representation to other representations, F3 is the skill of translating from algebraic representation to other representations, and F4 is the skill of translating from table representation to other representation.

There are some index types used in the literature to reveal whether the model examined in CFA fit to the data or not. They are (x^2)/sd, CFI, RMSEA, GFI, AGFI, NFI, NNFI and SRMR indices (Marsh & Hocevar, 1988). The values obtained from MRTMT, which are evaluated according to these indices, are given in the table below.

Table 5. *Fit values regarding MRTMT*

Fit indices	$\frac{X^2}{sd}$	CFI	RMSEA	GFI	AGFI	NFI	NNFI	SRMR
Model Values	304.27/ 64	0.99	0.063	0.96	0.93	0.98	0.98	0.032
Perfect Fit	$\frac{x^2}{sd} < 3$	0.97≤CFI ≤1	0<RMSEA<0.05	0.95≤GFI ≤1	0.90≤AGFI≤1	0.95≤NFI ≤1	0.95≤NNFI≤1	0≤SRMR ≤0.05

Acceptable Fit	$\frac{x^2}{sd} < 5$	0.90≤CFI ≤0.95 or 0.80≤GFI ≤0.89	0.05<RMSEA<0.10	0.90≤GFI ≤0.95 or 0.85≤GFI ≤0.90	0.85≤AGFI≤0.90	0.90≤NFI ≤0.95	0.90≤NFI ≤0.95	0.05≤SRMR≤0.10

According to Table 5, chi square /sd (4.75) and RMSEA (.063) fit indices are in acceptable fit range and other indices of CFI (.99), GFI (.96), AGFI (.93), NFI (.98), NNFI (.98), SRMR (.032) show perfect fit. Factor loads are seen in the figure; it ranges from .78 to 1.12 for the F1 sub-dimension, .72 to .87 for the F2 sub-dimension, .65 to .82 for the F3 sub-dimension, and .27 to .95 for the F4 sub-dimension. It was understood that there was no problem in the observed t values and factor load values. Jöreskog and Sörbom (1996), states that, the absence of red arrows in the t values indicates that all items are significant at the .05. No items had to be removed from the test.

Thus, the findings of Rasch analysis and the values obtained from CFA reveal that the developed MRTMT is a valid and reliable instrument.

3.3. Data Analysis

Descriptive statistics (mean, standard deviation) were used to explain the level of multiple representation translate skills of 9th grade students. Accordingly, SPSS 21.00 program was used. In addition, to answer the research question “Are there relationships among 9th grade students’ verbal- graph- algebraic- table multiple representation skills in algebra learning area?”, correlation statistics were run and the data were analyzed through SPSS 21.0. The repeated measures ANOVA was performed to explain the significant differences between factors.

Quantitative data consist of responses obtained from 12 open ended questions in the MRTMT. Analytical scoring key developed for the measurement tool is presented in Table 6.

Table 6. *Analytical Scoring Key*

Score	Explanation
0 point	No solution path or wrong solution path
	No result or wrong result
1 point	Solution path is partially correct, result is wrong
	Modeling or mathematical sentence is partially correct, result is wrong
2 points	Solution path is correct, result is wrong
	No modeling or mathematical sentence, result is correct
3 points	Solution path is correct, result is correct
	Correct modeling or mathematical sentence, result is correct

(Cetin, & Ertekin, 2011)

The highest score that can be obtained from the scale is 36 and the lowest score is 0. Within the scope of the analysis, mean and standard deviation values were examined in order to obtain descriptive statistics of the data.

On the other hand, the correlations were examined separately in order to reveal the relationships between students’ translate skills of graph, table, verbal and algebraic representations in the field of algebra. The relationship between students' ability to translate between representations was determined by calculating the "Pearson Correlation" coefficients based on the relationships between the verbal-algebraic, verbal-table, verbal-graph, algebraic-table, algebraic-graph, table-graph representation forms which are present in each test part.

4. Findings

To answer the research question "What is 9th grade students' level of multiple representation translate skills in algebra learning area?", means and standard deviations of translate between representations were calculated and the results are provided in Table 7.

Table 7. *Means and Standard Deviations of Factors*

Translate Between Representations	**N**	**$\bar{X}$**	**SD**
Translate from Verbal Representation to Other Representations	637	2.88	3.10
Translate from Graph Representation to Other Representations	637	3.15	2.99
Translate from Algebraic Representation to Other Representations	637	1.89	2.44
Translate from Table Representation to Other Representations	637	3.31	2.63

(min=0, max=9)

According to Table 7, the value of translating from verbal representation to other representations is ($\bar{X}$=2.88; SD =3.10), the value of translating from graph representation to other representations is ($\bar{X}$=3.15; SD =2.99), c the value of translating from algebraic representation to other representations is ($\bar{X}$=1.89; SD =2.44), t the value of translating from table representation to other representations is ($\bar{X}$=3.31; SD =2.63).

According to the findings of descriptive statistics performed in order to identify 9th grade students' multiple representations translate skills in algebra, as provided in Table 7, students are more successful in translating from table representation to other representations compared to other representation translates. Another translate that student generally succeed in is translating from graph representation to other representations, while students have difficulty in translating from algebraic representation to other representations.

In order to identify whether there is a statistically significant difference between multiple representations translate score means in the factors or not, repeated measures ANOVA test was performed. Since the sphericity assumption, which is one of the assumptions of this test, was not met, the F value obtained from the Greenhouse and Geisser correction (cited in: Abdi, 2010) was used in the reporting. The results of this test, which is conducted to determine whether the difference in scores between factors is statistically significant, are provided in Table 8.

Table 8. *ANOVA test findings*

<table>
<tr><th>Variables</th><th>Variance Source</th><th>Sum of Squares</th><th>df</th><th>Mean squares</th><th>F</th><th>p</th><th>η^2</th><th>Significant Difference</th></tr>
<tr><td rowspan="2">Translate between Multiple Representations</td><td>Measurement</td><td>778.049</td><td>2.810</td><td>276.927</td><td rowspan="2">72.066</td><td rowspan="2">0.00</td><td rowspan="2">0.286</td><td rowspan="2">2>1, 1>3, 4>1, 2>3, 4>3</td></tr>
<tr><td>Error</td><td>6866.451</td><td>1786.895</td><td>3.843</td></tr>
</table>

1: translate from verbal representation to other representations 2: translate from graph representation to other representations 3: translate from algebraic representation to other representations 4: translate from table representation to other representations

There is a statistically significant difference between the mean scores obtained by 9th grade students from factors of MRTMT [$F(2.81; 1786.89) = 72.066$, $p<.01$, $\eta^2=.286$). It is seen that the effect size value is high (Cohen, 1988).

Then, comparisons were made with the paired t test in order to see the differences in success in detail and it was seen that there is a significant difference between all factors except the skills of translating from graph representation to other representations and the skill of translating from table representation to other representations. While the significant difference between students' translate skill from algebraic representation to other representations and their translate skill from table representation to other representations is the greatest (4> 3), the difference between their translate skill from verbal representation to other representations and their translate skill from graph representation to other representations is the smallest (2> 1).

As a result of the descriptive analysis, the means and standard deviations for each item of the factors are presented in Table 9.

Table 9. *Means and standard deviations of each item*

	Translate from verbal representation			Translate from graph representation			Translate from algebraic representation			Translate from table representation		
	Algebraic(M1)	Table (M2)	Graph (M3)	Verbal (M4)	Algebraic (M5)	Table (M6)	Verbal (M7)	Table (M8)	Graph (M9)	Verbal (M10)	Algebraic (M11)	Graph (M12)
$\bar{X}$	1.11	.82	.96	1.67	.42	1.06	.56	.75	.58	1.32	.74	1.25
S.d	1.08	1.23	1.19	1.37	.97	1.33	.98	.97	.87	1.24	1.11	1.28

(min=0, max=3), N=637

According to Table 9, the value of translate from verbal representation to algebraic representation is ($\bar{X}$=1.11; SD =1.08), to table representation is ($\bar{X}$=.82; SD =1.23), to graph representation is ($\bar{X}$=.96; SD =1.19). While translating from verbal representation, the item that students are most successful in is item 1, which requires translate to algebraic representation, while the item they have the most difficulty is item 3, which requires translate to graph representation.

According to Table 9, the value of translate form graph representation to verbal representation is ($\bar{X}$=1.67; SD =1.37), to algebraic representation is ($\bar{X}$=.42; SD = .97), to table representation is ($\bar{X}$=1.06; SD = 1.33). While translating from graph representation, the item that students were most successful in was the 4th item that requires translate to verbal representation, while the item they had the most difficulty with was the 6th item that required translate to the table representation.

The value of translate from algebraic representation to verbal representation is ($\bar{X}$=.56; SD =.98), to table representation is ($\bar{X}$=.75; SD =.97), to graph representation is ($\bar{X}$= .58; SD =.87). While translating from algebraic representation, the item that students are most successful in is the 7th item, which requires translate to the table representation, the item they had the most difficulty with was the 9th item that requires the translate to verbal representation.

The value of translate from table representation to verbal representation is ($\bar{X}$=1.32; SD =1.24), to algebraic representation is ($\bar{X}$=.74; SD =1.11), to graph representation is ($\bar{X}$=1.25; SD =1.28). While translating from table representation, the item that students were most successful in was item 10, which requires translate to verbal representation, the item they had the most difficulty with was the 11th item, which requires translate to algebraic representation.

In general, it is seen that the translate form in which students are most successful when translating between representations in the equations and disequilibrium sub-learning area is translate from graph representation to verbal representation (M1), while the translate form in which they most unsuccessful is translate from algebraic representation to graph representation (M9).

To answer the research question “Are there relationships among 9th grade students’ verbal- graph-algebraic- table multiple representation skills in algebra learning area?”, Pearson Correlation coefficients were calculated for each representations through translate scores and the results are provided in Table 10.

Table 10. *Pearson Correlation Analysis Results*

	Verbal	Algebraic	Table	Graph
Verbal	1			
Algebraic	,523**	1		
Table	,603**	,702**	1	
Graph	,585**	,685**	,751**	1

** p< .01

Cohen (1988) states that the r value obtained in the correlation analysis indicates a low level relationship between .10 and .29, a moderate relationship between .30 and .49, and a high level between .50 and 1.0.

As a result of the Pearson correlation analysis, a significant and high level relationship was found between 9th grade students' skills of translating to algebraic representation and translating to verbal representation in the sub-learning area of equations and disequilibrium ($r=.52$, $p<.01$). Therefore, it can be suggested that as students' translate skills to algebraic representation increase, their translate skills to verbal representation will increase.

Besides, a significant and high level relationship was also found between 9th grade students' skills of translating to table representation and translating to verbal representation ($r=.60$, $p<.01$) and translating to algebraic representation ($r=.70$, $p<.01$) in the sub-learning area of equations and disequilibrium. Accordingly, it can be said that if students' translate skills to table representation increase, their translate skills to verbal and algebraic representations will increase.

Lastly, a significant and high level relationship was also found between 9th grade students' skills of translating to table representation and translating to verbal representation($r=.58$, $p<.01$), translating to algebraic representation ($r=.68$, $p<.01$) and translating to table representation ($r=.75$, $p<.01$) in the sub-learning area of equations and disequilibrium. Accordingly, it can be said that if students' translate skills to graph representation increase, their translate skills to verbal, algebraic and table representations will increase.

5. Discussion

An examination of the answers given by the students to the Multiple Representation Translating Measurement Tool, which was originally developed in the current study to investigate students' translate skills between multiple representations in algebra, reveals that students' skills of translating between representations are at a low level. This finding is in line with the studies in the literature (Akkus & Cakiroglu, 2006; Bossé et al., 2014; De Bock et al., 2015; Demir & Cansiz-Aktas, 2018; Gurbuz & Sahin, 2015; Ipek & Okumus, 2012; Kardes et al., 2012; Mourad, 2005; Sert, 2007).

The findings of the current study suggest that the translate of representation that students are most successful in is the translate from graph representation to verbal representation ($\bar{X}$ = 1.67), while the translate of the representation they have the most difficulty with is the translate from graph representation to algebraic representation ($\bar{X}$ = .42) and from algebraic representation to verbal representation ($\bar{X}$ = .56). Similarly, it is highlighted in the literature that the translate form that students are most successful with is transformation of graph-verbal (Andrá et al., 2015) and the translate form they have most difficulty with is transformation of verbal-algebraic (De Bock et al., 2015; Delice & Sevimli, 2010; Galbraith & Haines, 2000). On the other hand, although the students were quite successful in translating from graph representation to verbal representation, they were not so successful in the opposite direction. It is thought that this problem of the students may stem from the need for more than one representation translate such as symbolic, equation, schematic and numerical in the translate of verbal representation to graph representation, as stated by Raviwati et al (2017).

It is thought that the reason why students have difficulty in graph-algebraic and algebraic-graph representation translate is their difficulty in analyzing the effect of changing parameters (De Bock et al., 2015). In addition, when all representation translates are considered, the finding that students have the most difficulty in translating algebraic and graph representations is supported by the finding by Bossé et al., (2014) that students switch graph features into algebraic formulas by going through a similar translate path.

The second translate that students find most difficult is the translate from algebraic representation to verbal representation. This finding is in line with Wang's (2015) study. On the other hand, Gurbuz & Sahin's (2015) finding that 8th grade students have problems in translating from graph and algebraic representation to verbal representation in equations and disequilibrium sub-learning area is in parallel with the algebraic-verbal translate performance in our study yet it contradicts with graph-verbal translate performance. This situation can be explained by the difference in difficulty level of the translate problems between representations prepared in both studies.

On the other hand, the verbal representation is the representation that students are most successful when translating from graph representation. This result is in line with the studies in the relevant literature (Andrá et al., 2015; İncikabı, 2016; Rahmawati et al., 2017). In the study in which İncikabı (2016) examined the types of representations which were used in the questions in the middle school mathematics textbooks and which were needed to solve the questions, it was found that verbal representation was requested as a solution in 68% of the questions containing graphics in their expressions.

The finding that the representation that students have the most difficulty in translating from algebraic representation is the verbal representation is supported by Andrá et al. (2015). As Mourad (2005) states, algebraic representation can cause difficulties and confusion for students. A student who is not proficient in algebraic representation cannot distinguish between the uses of representation, causing confusion and misunderstanding of concepts. On the other hand, the finding that the representation that students are most successful in translating from algebraic representation is the table representation is supported by the study of Bossé et al. (2011). The finding that the representation that students have the most difficulty in translating from table representation is algebraic representation is supported by the relevant literature (Adu-Gyamfi et al., 2012; Presmeg & Nenduradu, 2005).

The finding that students are successful in translating from table representation to other representations is in parallel with the studies in the literature (Akkus & Cakiroglu, 2006; Bal, 2015). The effective use of table representation in many lessons can be shown as the reason why students are successful in translating from table representation to other representations (Bal, 2015).

It has been found that students have difficulty in translating from algebraic representation to other representations. Many studies in the relevant literature support this finding (Andrá et al., 2015; Deniz, 2016; Ozhan-Turan, 2011). It is stated that the type of stimulus that challenges students the most is formula stimuli (Andra et al., 2015).

The significant relationship determined between the students' graph representation translate skill in algebra and their ability to translate to verbal representation, expressed in the correlation analysis findings, is parallel to the finding of the study conducted by Adiguzel and Akpinar (2004) that success in finding and representing the graph representation increases the performance in algebra representation.

6. Conclusion

As a result, the students had the most difficulty in translating from other representations to algebraic representation and from algebraic representations to other representations. It has been observed that they generally cannot distinguish between table and graph representation.

When students learn certain algorithms, they can do operations (algebraic) quite easily. However, given an algebraic equation, they cannot illustrate the situation and interpret it from different aspects. In other words, the deficiency in understanding a mathematical sentence and explaining it with other representations also shows that they cannot learn conceptually and in a qualified way. For instance, as well as obtaining and solving the $y = 2x$ equation, the student is expected to be able to explain what this mathematical sentence expresses with different representations suitable for a linear equation. In algebra teaching, solving equations successfully is possible through having students attain meaningful and conceptual learning by establishing mathematical relationships in also real-life situations as well as getting them to find the unknown. Therefore, the translate of representations to each other has a very critical place in algebra teaching.

According to another result of the study, there is a significant relationship between the verbal-graph-algebraic-table representation translate skills of the students. Based on these findings, it can be said that more emphasis should be placed on the use of representation in mathematics lessons. Therefore, teachers should encourage students to use representation and translate between representations. The idea that the use of multiple representations facilitates students' problem solving is supported by many studies (Deniz, 2016; Fonger et al., 2018; Huntley & Davis, 2008; Parrot & Leong, 2018; Sarıhan-Musan, 2012).

7. Recommendation

For the practitioners, it can be suggested that they should use multiple representations in classroom practices; students' expressing their thoughts should be encouraged through in-class discussion to let them understand how translating between representations are made; technology supported materials ensuring multiple thinking should be made use of rather than using only abstract symbolic language in algebra teaching.

For researchers, it can be suggested that they may examine the effects of different teaching methods in mathematics education on multiple representations translate skills; qualitative studies can be carried out regarding why multiple representations translate skills levels of students at various grades are low and why have difficulty in this; MRTMT, developed in this study for 9th grade students, can be used and its relationship with different learning areas/skills/performances can be examined. In addition, it can be suggested to investigate why students have difficulty in translating to some representation types while not having difficulty in some other representation types in terms of variables such as thinking structures, learning styles, etc.

Limitations

The research is limited to the descriptive findings of translating multiple representations obtained by the quantitative research method in the Equations and Inequalities subject in algebra unit.

References

Abdi, H. (2010). The greenhouse-geisser correction. *Encyclopedia of research design*, 1, 544-548.

Adigüzel, T., & Akpinar, Y. (2004). Improving School Children's Mathematical Word Problem Solving Skills through Computer-Based Multiple Representations. *Association for Educational Communications and Technology; 27th, Chicago, IL, October 19-23*.

Adu-Gyamfi, K., Stiff, L. V., & Bossé, M. J. (2012). Lost in Translation: Examining Translation Errors Associated with Mathematical Representations. *School Science and Mathematics*, 112(3), 159–170. doi:10.1111/j.1949-8594.2011.00129.x

Akkus, O., & Cakiroglu, E. (2006). Seventh grade students' use of multiple representations in pattern related algebra tasks. *Hacettepe University Journal of Education*, 31(31), 13–24.

Andrá, C., Lindström, P., Arzarello, F., Holmqvist, K., Robutti, O., & Sabena, C. (2015). Reading mathematics representations: An eye-tracking study. *International Journal of Science and Mathematics Education*, *13*(2), 237-259.

Aviles-Garay, E. J. (2001). *Usıng Multıple Coordınated Representatıons in a Technology? IntensıveSettıng to Teach LınearFunctıons at the College Level*. Doctoral Thesis, Urbana.

Baghaei, P., & Amrahi, N. (2011). Validation of a multiple choice English vocabulary test with the rasch model. *Journal of Language Teaching & Research,* 2(5), 1052- 1060.

Baki, A. (2018). *Matematiği Öğretme Bilgisi [Knowledge of Teaching Mathematics]*. Pegem Akademi, Ankara.

Bal, A. P. (2015). Skills of Using and Transform Multiple Representations of the Prospective Teachers. *Procedia - Social and Behavioral Sciences*, 197(February), 582–588. doi:10.1016/j.sbspro.2015.07.197.

Bayik, F. (2010). *11. Sınıf Öğrencilerinin Geometrik Problemlerle İlgili Oluşturdukları Dış Temsillerle İç Temsiller Arasındaki Etkileşimler [Interactions Between Internal Representations and External Representations Generated By 11. Class Secondary Students' Related with Geometric Problems]*. Master Thesis, Institute of Education Sciences, Ankara.

Bieda, K. N., & Nathan, M. J. (2009). Representational Disfluency in Algebra: Evidence From Student Gestures And Speech. *Zdm - International Journal On Mathematics Education*, 41(5), 637–650.

Bond, T., & Fox, CM. (2015). *Applying the Rasch model: Fundamental Measurement in the Human Sciences.* London: Routledge.

Boone, W. J., Staver, J. R., & Yale, M. S. (2014). *Rasch Analysis in the Human Sciences.* Dordrecht: Springer.

Bossé, M. J., Adu-Gyamfi, K., & Cheetham, M. (2011a). Transitions Among Mathematical Representations: Teachers Beliefs and Practices. *International Journal of Mathematics Teaching and Learning*, 15(6), 1–23.

Bossé, M. J., Adu-Gyamfi, K., & Cheetham, M. R. (2011). Assessing the difficulty of mathematical translations: Synthesizing the literature and novel findings. *International Electronic Journal of Mathematics Education*, 6(3), 113–133.

Bossé, M., Adu-Gyamfi, K., & Chandler, K. (2014). Students' Differentiated Translation Processes. *International Journal for Mathematics Teaching and Learning*, (828).

Cetin, H., & Ertekin, E. (2011). The Relationship between Eighth Grade Primary School Students' Proportional Reasoning Skills and Success in Solving Equations. *International Journal of Instruction*, 4(1), 47-62.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*, 2nd ed. Hillsdale, NJ: Erlbaum.

De Bock, D., Van Dooren, W., & Verschaffel, L. (2015). Students' Understanding of Proportional, Inverse Proportional, and Affine Functions: Two Studies on the Role of External Representations. *International Journal of Science and Mathematics Education*, 13(1), 47–69. doi:10.1007/s10763-013-9475-z

Delice, A., & Sevimli, E. (2010). Öğretmen adaylarının çoklu temsil kullanma becerilerinin problem çözme başarıları yönüyle incelenmesi: Belirli integral örneği [An investigation of the pre-services teachers' ability of using multiple representations in problem-solving success: The case of definite integral]. *Educational Sciences: Teory& Practice*, *10*(1), 111–149.

Demir, Ö., & Cansız-Aktas, M. (2019). *8. Sınıf Öğrencilerinin Çoklu Temsiller Arasındaki Geçiş Becerileri: Doğrusal İlişki İçeren Durumlar Örneği [8th Grade Students' Skills in Translating Among Multiple Representations]*. 4th International Symposium of Turkish Computer And Mathematics Education. İzmir.

Deniz, S. (2016). *Doğrusal Denklemlerin 7. Sınıflarda Öğretiminde Geometri Sketchpad Kullanımının Çoklu Temsil Ve Enstrümantal Yaklaşım Boyutundan İncelenmesi [Examination of the use of geometry sketchpad with respect to multiple representations and instrumental approach while teaching linear equations in 7th grade]*. Master Thesis, Anadolu University Institute of Educational Sciences, Eskişehir.

Dervent, F., Devrilmez, E., İnce, M. L., & Ward, P. (2018). *Beden eğitimi öğretmenleri için futbol genel alan bilgisi bilgi testinin güvenirlik ve geçerliği [Reliability and Validity of Football Common Content Knowledge Test for Physical Education Teachers]*. *Hacettepe Journal of Sport Science*, 29(1), 39-52

Deutsche Kultus minister konferenz [KMK]. (2003). *BildungsstandardsimFachMathematikfür den MittlerenSchulabschluss*. München: Luchterhand.

Dreher, A., Kuntze, S., & Lerman, S. (2015). Why Use Multiple Representations in the Mathematics Classroom? Views of English and German Preservice Teachers. *International Journal of Science and Mathematics Education*, 14, 363–382. doi:10.1007/s10763-015-9633-6

Dundar, S. (2015). Mathematics Teacher- Candidates' Performance in Solving Problems with Different Representation Styles: The Trigonometry Example. *Eurasia Journal of Mathematics, Science & Technology Education*, 11(6), 1379–1397. doi:10.12973/eurasia.2015.1396a

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1–2), 103–131. doi:10.1007/s10649-006-0400-z

Egodawatte, G. (2011). *Secondary School Students' Mısconceptıons In Algebra*. Doctoral Thesis, Department of Curriculum, Teaching and Learning, University of Toronto.

Even, R. (1998). Factors involved in linking representations of functions. *Journal of Mathematical Behavior*, 17(1), 105–121. doi:10.1016/s0732-3123(99)80063-7

Fonger, N. L., Davis, J. D., & Rohwer, M. Lou. (2018). Instructional Supports for Representational Fluency in Solving Linear Equations with Computer Algebra Systems and Paper-and-Pencil. *School Science and Mathematics*, 118(1–2), 30–42. doi:10.1111/ssm.12256.

Gagatsis, A., & Shiakalli, M. (2004). Ability to translate from one representation of the concept of function to another and mathematical problem solving. *Educational Psychology*, 24(5), 645–657. doi:10.1080/0144341042000262953

Galbraith, P., & Haines, C. (2000). Conceptual mis(understandings) of beginning undergraduates. *International Journal of Mathematical Education in Science and Technology*, 31(5), 651–678. doi:10.1080/002073900434350.

Goldin, G. (2003). Representation in school mathematics: A unifying research perspective. *The National Council of Teachers of Mathematics*, (October).

Greeno, J. G., & Hall, R. P. (1997). Practicing Representation: Learning with and about Representational Forms. *Phi Delta Kappan*, 78(5), 361–367.

Gurbuz, R., & Sahin, S. (2015). 8. Sınıf Öğrencilerinin Çoklu Temsiller Arasındaki Geçiş Becerileri. *Kastamonu Education Journal*, 23(4), 1869–1888.

Heinze, A., Star, J. R., & Verschaffel, L. (2009). Flexible and adaptive use of strategies and representations in mathematics education. *ZDM - International Journal on Mathematics Education*, 41(5), 535–540. doi:10.1007/s11858-009-0214-4.

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. *Journal of Mathematical Behavior*, 17(1), 123–134. doi:10.1016/s0732-3123(99)80064-9

Hitt, F. (2002). Representation and Mathematics Visualization. *Journal of Chemical Information and Modeling (C. 53)*. doi:10.1017/CBO9781107415324.004.

Huntley, M. A., & Davis, J. D. (2008). High-School Students' Approaches to Solving Algebra Problems that are Posed Symbolically: Results from an Interview Study. *School Science and Mathematics*, 108(8), 380–388. doi:10.1111/j.1949-8594.2008.tb17852.x

İncikabı, S. (2016). *Ortaokul Matematik Ders Kitaplarının Farklı Temsilleri Kullanım Biçimlerinin Araştırılması [An investigation of middle school mathematics textbooks' inclusion of multiple representations]*. Master Thesis, Kastamonu University, Graduate School of Natural and Applied Sciences. Kastamonu.

Ipek, A. S., & Okumus, S. (2012). İlköğretim Matematik Öğretmen Adaylarının Matematiksel Problem Çözmede Kullandıkları Temsiller [The Representations of Pre-service Elementary Mathematics Teachers Used in Solving Mathematical Problems]. *Gaziantep University Journal of Social Sciences*, 11(3), 681–700.

Janvier, C. (1987). Conceptions and representations: The circle as an example. *Problems of representation in the teaching and learning of mathematics*, 147–158.

Jöreskog, K. G., & Sörbom, D. (1996). *LISREL 8: User's reference guide*. Scientific Software International.

Kalaivani, K., & Tarmizi, R. A. (2014). Assessing Thinking Skills: A Case of Problem-based learning In Learning of Algebra Among Malaysian Form Four Students. *Journal of International Academic Research for Multidisciplinary*, 2(3), 166–173.

Kalkan, B. N. (2014). *Cebir Öğrenme Alanında Çoklu Zekâ Kuramı Destekli Öğretimin Öğrencilerin Matematik Dersine Yönelik Tutumuna Etkisi [The Effect of Multiple Intelligence Theory Supported Instruction in the Field of Algebra Learning on Students' Attitudes Towards Mathematics Lesson].* National Science and Mathematics Education Congress, Adana.

Kaput, J. J. (1998). Representations, inscriptions, descriptions and learning: A kaleidoscope of windows. *The Journal of Mathematical Behavior*, 17(2), 265–281. doi:10.1016/s0364-0213(99)80062-7.

Kardes, D., Aydin, E., & Delice, A. (2012). *Öğretmen adaylarinin lineer denklem sistemleri konusundaki temsil dönüşüm başarilari [Representational transformation achievements of teacher candidates on linear equation systems.* X. National Science and Mathematics Education Congress, Niğde.

Kaya, D. (2015). Çoklu Temsil Temeli Öğretimin Öğrencilerin Cebirsel Muhakeme Becerilerine, Cebirsel Düşünme Düzeylerine ve Matematiğe Yönelik Tutumlarına Etkisi Üzerine Bir İnceleme [A study on the effects of multiple representations-based instruction on students' algebraic reasoning skills, algebraic thinking levels and attitudes towards mathematics]. Doctoral thesis, Dokuz Eylul University Institute of Educational Sciences, İzmir.

Keller, B. A., & Hirsch, C. R. (1998). Student preferences for representations of functions. *International Journal of Mathematical Education in Science and Technology*, 29(1), 1–17. doi:10.1080/0020739980290101

Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations Among Representations in Mathmatics Learning and Problem Solving. *Problems of Representation in the Teaching and Learning of Mathematics (Ed: In C. Janvier),* Hillsdale: NJ: Lawrence Erlabaum Associates.

Lew, H. (2004). Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics 1. *The Mathematics Educator*, 8(1), 88–106.

Linacre, J. M. (2008). *A User's Guide to Winsteps Rasch-Model Computer Program.* Version 3.65. Available at: www.winsteps.com (accessed 30 September 2019).

Marsh, H. W., & Hocevar, D. (1988). A New, More Powerful Approach To Multitrait-Multimethod Analyses: Application Of Second-Order Confirmatory Factor Analysis. *Journal of Applied Psychology*, 73, 107-117.

McKendree, J., Small, C., Stenning, K., & Conlon, T. (2002). The role of representation in teaching and learning critical thinking. *Educational Review*, 54(1), 57–67. doi:10.1080/00131910120110884.

Meydan, C. H., & Şeşen, H. (2016). *Yapısal Eşitlik Modellemesi AMOS Uygulamaları* [Structural Equation Modeling AMOS Applications]. Detay Publishing, 138, Ankara.

Milli Eğitim Bakanlığı- Ministry of National Education [MoNE] (2018). *Ortaöğretim Matematik Dersi (9, 10, 11 ve 12. Sınıflar) Öğretim Programı [Secondary School Mathematics (9th, 10th, 11th and 12th Grades) Curriculum].*

Mourad, N. M. (2005). *Inductive Reasoning in the Algebra Classroom,* Master Thesis, The Faculty Of The Department Of Mathematics San Jose State University. doi: 10.1016/J.Jaci.2012.05.050.

NCTM. (2000). Principles and Standards for School Mathematics. *National Council of Teachers of Mathematics*, 1–6.

Ozhan-Turan, A. (2011). *12. Sınıf Öğrencilerinin Analitik Geometrideki Temsil Geçişlerinin Krutetskii Düşünme Yapıları Bağlamında İncelenmesi; Doğruların Birbirine Göre Durumları [Investigation of 12th grade high school students based on the transition between different representations in analytic geometry in the context of Krutestskii?s thinking structures: Status of straight lines].* Master Thesis, Marmara University Institute of Educational Sciences, İstanbul.

Pape, S. J., & Tchoshanov, M. A. (2001). The Role of Representation(s) in Developing Mathematical Understanding. *Theory Into Practice*, *40*(2), 118–127. doi:10.1207/s15430421tip4002.

Parrot, M. A. S., & Leong, K. E. (2018). Impact of Using Graphing Calculator in Problem Solving. *International Electronic Journal of Mathematics Education*, 13(3), 139–148. doi:10.12973/iejme/2704.

Presmeg, N., & Nenduradu, R. (2005). *An Investigation of a Preservice Teacher'S Use of Representations in Solving Algebraic Problems Involving Exponential Relationships.* Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, 4, 105–112.

Rahmawati, D., Purwanto, P., Subanji, S., Hidayanto, E., & Anwar, R. B. (2017). Process of mathematical representation translation from verbal into graphic. *International Electronic Journal of Mathematics Education*, 12(4), 367–381.

Rasch, G. (1980). *Probabilistic Models for some Intelligence and Attainment Tests.* Chicago: University of Chicago Press.

Rau, M. A. (2017). Conditions for the Effectiveness of Multiple Visual Representations in Enhancing STEM Learning. *Educational Psychology Review*, 29(4), 717–761. doi:10.1007/s10648-016-9365-3

Robson, C. (2017). Tarama ve Anketler. *Bilimsel Araştırma Yöntemleri- Gerçek Dünya Araştırması [Screening and Surveys. Scientific Research Methods - Real World Research].* Editörler: Çınkır, Ş. ve Demirkasımoğlu, N. Anı Publishing, Ankara, 270-321.

Roth, W., & Bowen, G. M. (2001). Professionals Read Graphs : A Semiotic Analysis Linked references are available on JSTOR for this article : Professionals Read Graphs : A Semiotic Analysis. *Journal for Research in Mathematics Education*, 32(2), 159–194.

Sarihan-Musan, M. (2012). *Dinamik Matematik Yazilimi Destekli Ortamda 8. Sınıf Öğrencilerinin Denklem Eşitsizlikleri Anlama Seviyelerinin Solo Taksonomisine Göre İncelenmesi [Investigation Of 8th Grade Students' Understanding Levels Of The Equation And Ineqaulities According To The Solo Taxonomy In The Dynamic Mathematics Software Aided Environment].* Pamukkale University Graduate School of Natural and Applied Sciences, Denizli.

Sarpkaya-Aktas, G. (2019). Cebirin Tarihi. *Uygulama Örnekleriyle Cebirsel Düşünme ve Öğretimi [History of Algebra. Algebraic Thinking and Teaching with Application Examples],* Editör: Sarpkaya-Aktas, G. Pegem Akademi, Ankara. 1-22.

Schultz, J. E., & Waters, M. S. (2000). Why Representations? *Encyclopedia of Educational Reform and Dissent*, 93(6), 448–453. doi:10.4135/9781412957403.n297.

Sert, Ö. (2007). Eighth grade students' skills ın translating among different representations of algebraic concepts. *Yüksek Lisans Tezi, Middle East Technical University*, Ankara.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating Productive Mathematical Discussions: Five Practices for Helping Teachers Move Beyond Show and Tell. *Mathematical Thinking and Learning,* 10(4), 313–340. doi:10.1080/10986060802229675.

Ural, A. (2012). Fonksiyon Kavramı: Tanımsal Bilginin Kavramın Çoklu Temsillerine Translate Edilebilmesi ve Bazı Kavram Yanılgıları [Concept of Function: Transferring the Definitional Knowledge to Multiple Representations and Some Misconceptions]. *Pamukkale University Journal of Education,* 31(1), 93-105.

van Ameron, B. A. (2002). *Reinvention Of Early Algebra. Developmental Research On The Transition From Arithmetic To Algebra.* Thesis Utrecht University, Holland.

Waisman, I., Leikin, M., Shaul, S., & Leikin, R. (2014). Brain Activity Associated With Translation Between Graphical and Symbolic Representations of Functions in Generally Gifted and Excelling in Mathematics Adolescents. *International Journal of Science and Mathematics Education*, 12(3), 669–696. doi:10.1007/s10763-014-9513-5.

Wang, X. (2015). The Literature Review of Algebra Learning: Focusing on the Contributions to Students' Difficulties. *Creative Education*, 06(02), 144–153. doi:10.4236/ce.2015.62013.

Wilkie, K. J. (2016). Students' use of variables and multiple representations in generalizing functional relationships prior to secondary school. *Educational Studies in Mathematics*, 93(3), 333–361. doi:10.1007/s10649-016-9703-x

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching Algebra to Students with Learning Difficulties: An Investigation of an Explicit Instruction Model. *Learning Disabilities Research and Practice*, 18(2), 121–131. doi:10.1111/1540-5826.00068.

Authors

Sevcan MERCAN ERDOĞAN, Karamanoglu Mehmetbey University, Department of Mathematics and Science Education, Karaman (Turkey). E-mail: sevcanmercan189@gmail.com

Hatice ÇETİN, Necmettin Erbakan University, Department of Primary Education, Konya (Turkey). E-mail: haticebts@gmail.com

Kamil ARI, Selcuk University, Department of Mathematics and Science Education, Konya (Turkey). E-mail: kamil.ari@selcuk.edu.tr

Acknowledgement

There was no conflict of interest among the authors and the authors were ranked in terms of contribution level. This study is a product of the master thesis of the first author under the supervision of two other authors.

ANNEX. Multiple Representation Translating Measurement Tool

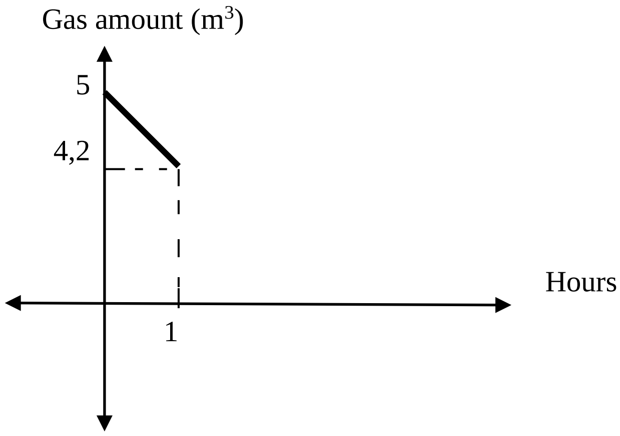
Multiple Representation Transfer Test

This measurement tool has been prepared for use in a scientific study. These questions will not be used to evaluate you. Read the questions carefully and write your answers clearly in the blanks.

Question 1: Ahmet, who likes to walk, went to the walking-track near his house at the weekend. He entered the track from the place where the 200 m inscription of the 3500 m long walking track is written. Since Ahmet's walking speed is 80 m/min:

1) Calculate the distance of Ahmet to the starting point of the track 5 minutes after he starts walking, by establishing an equation. Explain what the variables mean in the equation you have created.
2) Show the change in Ahmet's distance from the starting point of the track with a table. Please explain.
3) Draw a graph of Ahmet's distance (m) to the starting point of the track versus time (min).

Question 2: The variation of the gas in an oxygen supply cylinder over time is given below as a linear graph.



1) Explain the graph verbally.
2) Write an equation showing the amount of gas (m^3) in the oxygen supply cylinder over time (hours). What do the variables you use in the equation mean? Please explain.
3) Draw a table showing the amount of gas (m^3) in the oxygen supply cylinder over time (hours).

Question 3: In a dairy, milk is stored in cold cabinets with a capacity of 920 liters. On Monday, there are 50 liters of milk in the refrigerator. The change in the amount of milk according to the days is expressed as y= 50+30x. In this expression, x is the amount of milk obtained from the cow in a day, and the variable y is the amount of milk in the refrigerator.

1) Express the equation verbally.
2) Since a cow gives 6 liters of milk a day, show the amount of milk in the refrigerator at the end of each day with a table.
3) Since 6 liters of milk are taken from a cow per day, draw a graph showing the amount of milk over time until the refrigerator is full.

Question 4: In the table below, the scores of the students named Aylin, Batuhan, Can and Derya, who are studying at the same university, in the midterm and final exams from the mathematics course are given. In order to pass a course, at least 50 points must be obtained when 40% of the midterm exam and 60% of the final are added together.

	MIDTERM	FINAL
Aylin	65	55
Batuhan	90	80
Can	40	60
Derya	50	40

1) Express the above table verbally.
2) Write a general equation to calculate the score required for each student to pass the course. Explain what the variables mean in this equation.
3) Show the score obtained by Aylin, Batuhan, Can and Derya from the mathematics course with a graph.