Acta Didactica Napocensia

Volume 14, Number 2, 2021 - DOI: 10.24193/adn.14.2.7

Didactical Engineering in the Conception of a Teaching Situation Originated from Brazil´s SPAECE Assessment with the Support of the GEOGEBRA Software

Francisco Regis Vieira ALVES, Aline Maria da SILVA CAMILO, Francisca Cláudia Fernandes FONTENELE, Paula Maria Machado Cruz CATARINO

Abstract: This work presents a discussion about the Didactic Training Engineering (EDF) methodology, of French origin, emphasizing the role of the teacher, during his initial or continuous training, through the conception and structuring of a didactic situation, organized by such methodology and based on the Theory of Didactic Situations (TSD). Therefore, the objective of this work is to present a didactic situation, based on the research methodology of (EDF) and focused on the analysis of the teacher's role, through the application of the dialectical phases (action, formulation, validation and institutionalization) provided by the TSD. The mathematical problem chosen to compose such a situation, through preliminary and a priori analyzes by EDF, will be focused on Flat Geometry directed to the Permanent Evaluation System of Basic Education in Ceará (SPAECE). SPAECE (Brazil) is a large-scale assessment, held annually in the state of Ceará, Brazil, involving all elementary and high school students in public schools. In addition, the use of the GeoGebra software as a technological resource for carrying out a didactic transposition of the content covered in the proposed didactic situation stands out. For a better conduct of this teaching and learning process, it is suggested the adoption of a didactic contract between the characters involved and the consideration of possible epistemological obstacles and obstacles that may arise during this process.

Key words: Teacher training; Didactic Situation; GeoGebra; Teaching and learning.

1. Introduction

The growing interest in investigating the role of teacher education in Brazil is evidenced by the large number of scientific papers published on this topic (in conference proceedings, journals and books) in the last decades. Almeida and Nardi (2013), for example, report that at the last National Research Meeting in Science Education (VIII ENPEC), held in 2011, of the 1,235 papers approved for presentation and distributed in 14 investigative lines, 31% were related to the line teacher training. Much of such scientific work has shown an interest in systematic research and the examination of methodologies and didactic resources that assist teaching praxis within the scope of their profession in the classroom and in the school context.

In this sense, it is worth remembering that the activity of the mathematics teacher cannot be devoid of the adoption of certain assumptions that contribute to the understanding of didactic transposition or of a set of necessary changes in the scientific mathematical knowledge aimed at the school context and the learning of students. Consequently, the use of teaching methodologies for Mathematics in Brazil.

Therefore, in an attempt to provide another significant contribution to this issue, this work aims to present the assumptions of the research methodology, called Didactic Engineering of Training (EDF), which shifted from Classical Didactic Engineering (ED) and, in its essence, promotes the development

Received September 2020.

Cite as: Alves, F.R.V., Silva Camilo, A.M., Fontenele, F.C.F. & Catarino, P.M.M.C. (2021). Didactical Engineering in the Conception of a Teaching Situation Originated from Brazil´s SPAECE Assessment with the Support of the GEOGEBRA Software, *Acta Didactica Napocensia*, 84-98, https://doi.org/10.24193/adn.14.2.7

of resources, education and training for teachers of regular education, assisting the math teacher, in exercise, as well as the teachers in training, in the conception and application of learning situations in the teaching of mathematics (Perrin Glorian; Bellemain, 2019).

Originating in this context, the objective of this article also focuses on the presentation of a didactic situation, created from a problem situation, structured by EDF, as a research methodology, and developed from the perspective of the dialectical phases of Didactic Situations Theory (TSD) by Guy Brousseau (1976, 2002, 2008), a notion originally from a french research, in order to become useful for application in the classroom, by the teacher, aiming at improving the learning of his students and better repercussion of a mathematical culture in the classroom for the schools in Brazil.

In addition, such a didactic situation will be supported by the visual resources of the GeoGebra software, seen as a facilitator of the visual and intuitive perception of the mathematical elements, since this software allows the creation and dynamic manipulation of work, thus providing a better understanding during the resolution the problem situation.

2. Didactical Engineering for Research

The "Didactic Engineering of Training, Development or 2nd Generation Engineering" (Alves; Catarino, 2017, p. 126), concerns a qualitative research methodology, focused on training (initial or continued) presented as a resource for teacher education and training and named second generation, because it is based on a first classical didactic engineering, but prolonging the questioning. (Perrin-Glorian; Bellemain, 2019).

An immediate implication of interest, for the case of Didactic Development Engineering or Didactic Training Engineering (EDF) is that the constructs of an (ED), per se, become a corpus of knowledge for teachers and their necessary dissemination among other teaching professionals, with the potential to affect and modify certain scripts for a didactic transposition (Chevallard, 1991) and, consequently, a planned and systematic professional transposition.

Therefore, a brief discussion about this engineering will be presented below, while also addressing Classical or first generation Didactic Engineering.

2. 1. Some historical elements

The french research in Didactics of Mathematics showed interest in developing theoretical and conceptual frameworks specific to mathematics. In the early 1980s, Didactic Engineering (ED), also known as Didactic Engineering 1st generation, Classical Didactic Engineering or Didactic Engineering for Research (Almouloud; Silva, 2012), which, according to Alves (2018), has as main research interest the student's action, his development in learning and the classic principles based on the trinomial student - teacher - know. Artigue (1988), on the other hand, characterizes Didactic Engineering (classical) as an experimental scheme, based on didactic actions in the classroom, that is, on the conception, realization, observation and analysis of teaching sequence.

On the other hand, despite an endogenous diachronic process of DE, in the last years of the 1980s, discussions started to arise related to different points of view, on the part of didactics of mathematics, regarding Didactic Engineering. Regarding such debates, Perrin-Glorian and Bellemain (2019) state that learning situations were investigated, from an adidactic point of view, without studying the role of the teacher, even knowing that this is substantial in the return (action taken by the teacher when checking to the student the responsibility of a learning situation or a problem, where the student himself bears the consequences of this action) (Brousseau, 2008), in institutionalization (act by which the student takes into account, in an official way, the object of the knowledge and the teacher considers his learning) (ibid.) or in the development of tool-object dialectics and board games.

Such didactic situations were developed by experienced, highly qualified teachers, and associated with research, however after this stage, "didactic engineering has specifically become research methodologies" (Almouloud & Silva, 2012, p. 28).

Almouloud and Silva (2012, p. 23), express that "there is a very active didactic engineering, which is the result of a respectable evaluation, but generally refrains from providing precise analyzes and justifications that could enlighten users". Perrin-Glorian and Bellemain (2019), for their part, affirm that the teaching role was not the object of research among the first works related to Didactic Engineering, since the theory did not give space for such, probably because it is subtended, to At that time, the use of such didactic situations would be automatically available to teachers, at least to more experienced Mathematics teachers.

Thus, at the end of the 80s, there was a change in research, directed to Didactic Engineering, named Didactic Engineering of Training (EDF), whose greatest interest was shown to the mathematics teacher. In this sense, Almouloud and Silva (2012) explain that:

> The didactic development engineering is, according to Perrin-Glorian (2009), at the same time a didactic engineering for the development of resources and for the training of teachers involved in the project. The size of engineering is an important issue for development engineering and resource production. An isolated situation can be easily developed, but it cannot be expected to have a positive effect on the practice of teachers, in fact this type of situation can sometimes have a negative effect on the teaching and learning processes of mathematical concepts. Development engineering is strongly linked to investigations into the mathematical knowledge needed by teachers to teach mathematics. It is in this sense that it is linked to training (Almouloud; Silva, 2012, p. 32).

The following figures show some characteristics of the two engineering areas, with the interest of transmitting to the reader their similarities and differences, making a brief comparison to the 1st generation Didactic Engineering, which is also called Didactic Engineering for Research (IDR) and the 2nd generation Didactic Engineering, also called Development Didactic Engineering (IDD):

Table 1. ***Didactic Engineering and its objectives and aspects***

	Main goals	**General Aspect**
ED 1ª generation	Develop and study didactic transposition proposals for teaching.	Research methodology and with interest in the product.
ED 2ª generation	Determine the principles that govern the engineering that is to be transformed into a resource for regular education, and study the conditions for its dissemination.	Three non-independent functions: research, development and teacher training through analysis. It needs several levels of construction.

Source: Almouloud; Silva (2012, p. 46)

Table 2. ***Engineering for research and Engineering for development***

Didactic Engineering of 1st and 2nd generation	
IDR	IDD
• It raises didactic phenomena to study them; • It aims at advancing the research result, making use of experiments set up according to the research question; • There is no immediate concern to publicize the situations used.	• Produce resource (s) for teachers or for teacher training; • Freedom of action for the teacher; • Research remains essential, but research questions are not primarily motivated by the expansion of theoretical frameworks; • It is based on 1st generation engineering.

Source: Almouloud; Silva (2012, p. 46).

Undoubtedly, from the two tables and the arguments indicated in the preceding paragraphs, there is a strong interest in the observation of teaching praxis and a great commitment in the practice of teacher training in relation to the objectives of an EDF. In the following topic, some implications and repercussions on such research methodology will be addressed, as well as a description of the initial stages that underlie this work.

2. 2. Application and description of Didactic Engineering stages

As already mentioned, EDF has, at its origin, the objective centered on resources and professional training aimed at regular mathematics teachers. Such resources (or learning object), used to mediate teaching in the classroom, are realized through learning situations, in which they are conducted and organized by the teacher / researcher to improve the learning of their students. Thus, within the perspective of Didactic Development Engineering, it becomes necessary to provide for the adequacy of such situations and methods to conduct them, making it possible to make the decisions of the teacher more flexible, who, at the time, will have full responsibility for teaching in the classroom (Almouloud; Silva, 2012).

The following are three conditions for successful development of Didactic Development Engineering:

> 1. Leave a certain freedom of action to the teacher: this condition is already valid at the first level, but now it is a question of defining the sequence of situations with the teacher and analyzing how the teacher adapts the document provided to him.
>
> 2. Using the produced documents, teachers should try not to reproduce the story, but the conditions of learning, the essential question for didactic engineering, being how to identify the essential elements for the effective performance of the activity.
>
> 3. It is necessary to rely on a first generation didactic engineering that allows the construction of a fundamental situation and its analysis (Perrin-Glorian, 2009 apud Almouloud; Silva, 2012).

The research question must consider the need felt by teachers and their questions on the subject, as well as the needs identified by researchers, which are not necessarily equal to those of teachers (Perrin-Glorian; Bellemain, 2019). In addition, cooperation between researchers, trainers and teachers (illustrated in Figure 2), weaves the links between two levels of questioning by EDF, as we see below.

> […] a first level, where it is mainly about testing the theoretical validity of situations at the epistemological and cognitive level and identifying the essential engineering choices [..] a second level about the current practices in teaching this content and identifying the students' needs (difficulties (learning difficulties) and teachers (teaching difficulties) comparing those identified by researchers with those expressed by teachers and the institution (Perrin-Glorian; Bellemain, 2019, p. 74).

In figure 1, just below, we can see a sketch proposed by Perrin-Glorian & Bellemain (2019). The authors indicate the multiple relationships involving teachers (professeurs), researcher (chercheur), trainer (le formateur), resources (ressources), consideration of teaching issues (questions des recherche) and, finally, its implementation in the classroom.

The internal rectangle, therefore, represents a set of work, involved in the conversion between research and teaching, in which it has a group extended to teachers who test the situations and provide feedback among those involved. The small group of researchers and coaches direct the device and prepare interim versions of the resource, in which they are tested by the teacher educators, before distributing them for application in the classroom (Perrin-Glorian, 2019). Arrow 1 represents the transformation of the research hypothesis into a working hypothesis, which are brought to evolve in collective work, represented by the arrows intrinsic to the rectangle, and which leads to modifying the research hypotheses (arrows 2 and 3) (Perrin-Glorian ; Bellemain, 2019). In addition, these arrows, intrinsic to the rectangle, organize the design of resources, whose objective is to create appropriate teaching sequences to be applied in regular education (Perrin-Glorian, 2019).

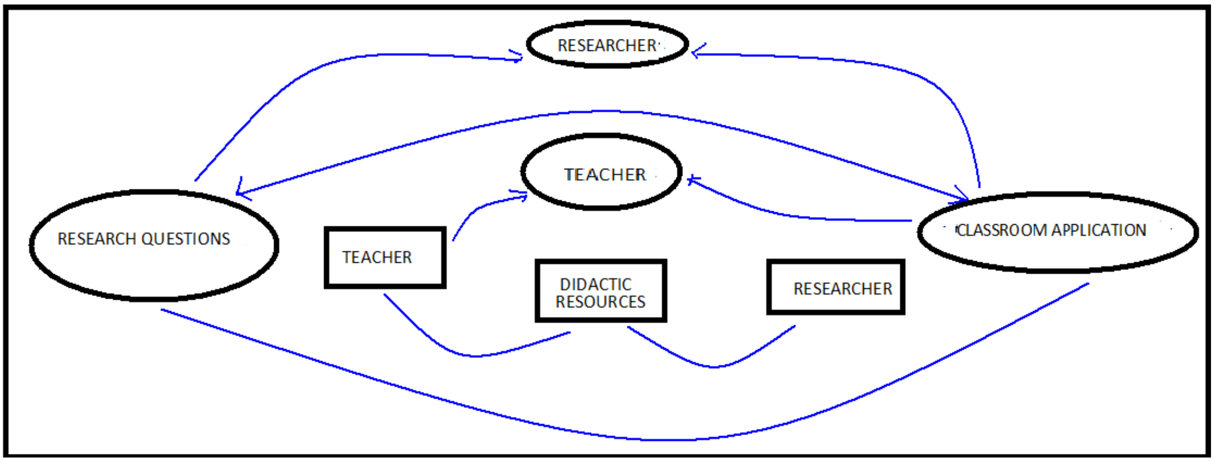

Figure 1. Organization of work between researchers, trainers and teachers

Because EDF is based on Classical Didactic Engineering, both go through the same four phases, namely: preliminary analyzes; a priori design and analysis; experimentation; and posterior analysis and validation. It is important to highlight that these phases do not necessarily follow the same order described here, and there may be, at times, articulation, anticipation or even overlapping of the elements described in these phases (Pommer, 2013).

Finally, all four phases will not be specified here, since, as it is a bibliographic research, with a qualitative approach, for a future master's research in Brazil, the objective of this work is to analyze only the construction of situations (not the implementation), based on the first two phases of EDF.

3. The Theoy of Didactical Situation (TDS)

In the previous section, mention was made of the development and design of teaching situations, based on the methodological assumptions of EDF. Therefore, in a character of theoretical complementarity, this work will use the Theory of Didactic Situations (TSD), as a way to conceive and structure such didactic situations aiming at its teaching.

The Theory of Didactic Situations (TSD) is a theoretical model, developed in France, by Guy Brousseau (1976, 2002, 2008), presented from a didactic situation, in which it provides the interaction between teacher, student and knowledge. Thus, the central object of study of TSD is not the cognitive subject, but the didactic situation, defined as follows:

> The set of relationships established explicitly and / or implicitly between a student or group of students, a certain milieu (possibly containing instruments or objects) and an educational system (the teacher) so that these students acquire a constituted or constituted knowledge (Brousseau, 1978 apud Almouloud, 2007, p. 33).

The student's learning takes place before his adaptation to a medium (milieu), which generates contradictions, difficulties and imbalances, regardless of the professor's intervention during the process (Brousseau, 2002). A means without didactic intentions is insufficient to provoke the desired knowledge in the student. Therefore, for the student to obtain this knowledge, it is necessary for the teacher to make a choice of problems, susceptible to learning, and to place them in front of the student, so that he can accept them and take a part of the responsibility for this learning, characterizing a return action (Brousseau, 2008).

There is also the adidactic situation, indicated by the moment when the student accepts the problem, until the moment when an answer is elaborated, without a direct and immediate intervention by the teacher in the answers about the knowledge he intends to institute, thus creating conditions for the student is the main character in the construction of his knowledge (Brousseau, 2008). Therefore, the prefix “a” indicates that the situation was temporarily free of its didactic intentionality (Artigue; Haspekian; Corblin-Lenfant, 2014).

This thorough selection of mathematical problems, made by the Mathematics teacher, as well as the means used in the transmission and transfer of knowledge involved in didactic situations, meets the idea of didactic transposition, characterized by Chevallard (1991) as the passage of knowledge wise to the taught knowledge.

Therefore, this theory is related to the proposed problem situation, in the figure of the teacher, since, when selecting the problem, he must interpret the knowledge (represented by scientific publications, textbooks, teaching manuals, etc.) and reformulate them in taught knowledge (represented by the teacher's role in preparing his class to be transmitted in the school environment), making necessary adaptations so that the student obtains the desired knowledge.

Brousseau (2002) relates a didactic situation to a game of student interactions with the environment (problem situation chosen by the teacher), thus representing a didactic relationship between the three elements involved: teacher, student and knowledge. Thus, the rule of this game is called a didactic contract, defined below:

> a relationship that determines - explicitly to some extent, but mainly implicitly - what each partner, teacher and student, will have the responsibility to manage and, in one way or another, be responsible to the other (Brousseau, 2002, p. 31).

It is also important to consider that, during the conduct of such situations, epistemological obstacles and obstacles may arise, that is, those that are “constitutive of the knowledge itself” (Brousseau, 2008, p. 51), and didactic, represented by difficulties teachers in transmitting certain knowledge (Brousseau, 2002).

Finally, Theory of Didactical Situation divides the process of interaction, or dialectics with the medium and formalization of mathematical knowledge, into four different phases. These phases or stages are indicated below:

a) dialectical situation of the action: “succession of interactions between the student and the environment” (Brousseau, 2002, p. 9). At the time, the student must outline strategies, using reasoning and his previous knowledge to formulate ways to solve the problem, without the intervention of the teacher;

b) dialectical situation of the formulation: moment in which the exchange of information takes place between the students themselves, using a language that is comprehensible to all involved (Brousseau, 2002);

c) dialectical situation of validation: situation in which the student must explain and prove the strategies created, submitting the mathematical message to the judgment of an interlocutor (Almouloud, 2007);

d) dialectical situation of institutionalization: moment when “the teacher conventionally and explicitly fixes the cognitive status of knowledge” (Almouloud, 2007, p. 40).

In the subsequent section, we will address and describe some elements necessary for the design of our didactic situation, aiming at teaching issues originated from SPAECE Brazil.

4. Representation of a didactic situation using the assumptions of EDF and TSD

In this section, the proposal of a didactic situation will be presented, through a mathematical problem of plane geometry aimed at SPAECE Brazil.

To this end, the first phase of EDF will be put into practice, since:

> The first phase is that in which preliminary analyzes are carried out, which may include the following aspects:
>
> • epistemological content aimed at teaching;

> • the usual teaching and its effects;
>
> • the students' conceptions, the difficulties and obstacles that mark their evolution;
>
> • the conditions and factors on which effective didactic construction depends;
>
> • consideration of specific research objectives;
>
> • the study of the didactic transposition of knowledge considering the educational system in which the work is inserted (Almouloud; Coutinho, 2008, p. 66).

Therefore, it is necessary to carry out analyzes on three important dimensions: epistemological, cognitive and institutional aspects (Perrin-Glorian; Bellemain, 2019). According to Brousseau (1997, 1998 apud Perrin-Glorian; Bellemain, 2019), the epistemological study, refers to the content itself, its structural possibilities, correlation with other contents, usually with a historical dimension.

The cognitive study, concerns the student's development, in the perspectives related to the content to be approached, and the difficulties already identified. Finally, the institutional study corresponds to the conditions for teaching the desired content, manifested through school programs, structuring teaching at class levels, at school or in the class itself, which can interfere with the implementation of situations.

From this perspective, SPAECE's high school mathematics pedagogical bulletins in the years 2013, 2015 and 2019 were analyzed, in which vast information was found regarding the exam history, student performance in the evaluation, reference matrix, scale proficiency, mastery of skills and competences, school results and suggestion of questions directed to evaluation.

This preliminary analysis made it possible to verify that, among the ten descriptors with the lowest percentage of correct answers, seven were related to Geometry, in the 2019 assessment, eight in the 2018 assessment and seven in the 2017 assessment (CEARÁ, 2019). Thus, it is possible to affirm that this poor performance of geometric knowledge, shown in the SPAECE results, is a reflection of the difficulty of understanding this subject in the classroom, since one of the objectives of large-scale assessments is to provide a diagnosis of the current condition. the level of proficiency acquired by students (Bertagna, Mello, Polato, 2014).

Therefore, in the face of the epistemological and didactic obstacles that involve Flat Geometry, it is suggested in this work to provide the teacher with an aid in its training process, through a didactic situation proposal that helps him to create teaching strategies directed to Geometry problems. Flat (focus of this study) in the classroom.

Then, the second phase of EDF was implemented, considered by Artigue (2014) as the crucial phase of the methodology, since it will be from this moment, after the investigation carried out in the previous step, that “the research hypotheses will be made explicit and engaged in the design of didactic situations ”(Artigue, 2014, p. 471).

Artigue (1988) differentiates between two types of command variables that will be conducted by the researcher: macrodidactic or global variables (which refer to the global engineering organization) and microdidactic or local variables (related to the local organization of an engineering, that is, the organization of didactic situations). In short, the objective of the a priori analysis is:

> [...] determine how the choices made allow you to control the student's behavior and meaning. Therefore, it will be based on hypotheses and these are hypotheses whose validation will, in principle, be indirectly involved in the confrontation carried out in the fourth phase between a priori and a posteriori analysis (Artigue, 1988, p. 294).

Thus, the choice of the problem situation, according to the criteria listed above, the use of the GeoGebra software as a teaching resource and the adoption of some assumptions of the Theory of Didactic Situations (and other theories that complement it), represent the choice of the macrodidactic variables that, thus, allowed “the characterization and design of the didactic sequence” (Pommer, 2013, p. 31).

According to Perrin-Glorian and Bellemain (2019), didactic situations should allow “the organization of mathematics, student learning opportunities and teaching conditions of teachers” (Perrin-Glorian; Bellemain, 2019, p. 48). In this perspective, from the SPAECE pedagogical Brazilian bulletins mentioned above, some mathematical problems present in such materials were analyzed, in which those that enabled the construction of a didactic situation that explored the assumptions of the TSD were selected, allowing the teacher to have a greater control of the actions performed by students, in addition to awakening actions that develop mathematical skills. In addition, this situation should make the application of GeoGebra viable, as a facilitator of the visual perception of the mathematical elements involved.

4. 1. The Didactical situation proposed with the support of GeoGebra software

The exposed problem involves the SPAECE D51 descriptor - “Solve the problem using the properties of the polygons (sum of the internal angles, number of diagonals and calculation of the internal angle of regular polygons)” (CEARÁ, 2013). It is true that the content of the properties of the polygons is much broader than that proposed in this question, but this selection of the subject, to be treated in the classroom, meets the idea of didactic transposition, indicated by Ramírez Bravo (2005) when state that:

> selection is not a matter of removing and placing, but an operation that sifts the content according to the socio-political and academic teaching and learning conditions. Not everything that exists needs to be taught, and it is in this context that the teacher fulfills the role of mediator (Ramirez & Bravo, 2005, p. 37).

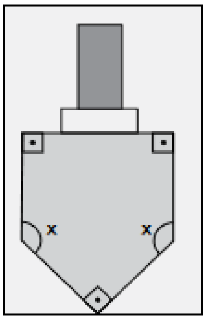

Figure 2. *Image removed from the 2015 SPAECE high school math pedagogical bulletin*

The drawing below represents a medal, in pentagonal format, manufactured to reward the players of a football tournament. What is the measure of the angle x in this drawing?

In addition, it is important to consider the epistemological and didactic obstacles present here, since Geometry, by itself, already represents an obstacle to student learning, as this content is generally seen with low frequency in the classroom compared to the Algebra, for example. This little coexistence with this subject ends up bringing, by students and even teachers, a feeling of aversion and insecurity to the theme. One of the reasons for the lack of familiarity with Geometry, part of the action of a large part of the teachers themselves, who, when they feel insecure to work with this subject, end up reserving little time in the school year to address it or simply ignore it (Pavanello, 1993).

Another aggravating aspect is the difficulty in exploring the student's visual perception, having few resources for that purpose, causing the teacher to choose to present ready-made formulas, even if their application does not offer real meaning in the student's daily experience. and consequently do not meet your expectations.

Therefore, it is important that the teacher organizes the overcoming of an obstacle, proposing a situation that can make the student progress, according to an appropriate interaction, allowing, from the beginning, the elaboration of a first solution or an attempt in which the student invests his knowledge of the moment.

> If this attempt fails or does not fit well, the situation must, however, return to a new situation modified by this failure in an intelligible but intrinsic way, that is,

> not depending on the arbitrary way of the master's purposes. The situation should allow for repeated testing of all student resources. It must be self-motivated by a subtle game of intrinsic sanctions (and not extrinsic sanctions linked by the teacher to the student's progress) (Brousseau, 1976, p. 109).

In other words, the teacher must reorganize the situation, if the student finds the presence of an obstacle, in view of the initial situation, readapting it, in order to explore all the student's cognitive resources, recognizing these obstacles, so that together they can “integrate their denial of learning new knowledge ”(Brousseau, 2008, p. 50).

It is important to note that in the course of the situation, there will certainly be insecurity and greed in some students, who will certainly try to ask for your help to find the answer right away. However, if there was a selection of the problem, according to the instructions mentioned in the previous excerpts, considering the epistemological and cognitive aspects of the target audience, the teacher must be attentive and avoid impulsive actions, caused by the teacher's desire to make the student learns what is being transmitted.

Therefore, the teacher must detach himself from traces of old and routine contracts, such as, for example, intervening during the student's interaction process with the environment, which may lead to the risk of influencing the design of his own fundamentals and strategies, such as commented on in the following excerpt:

> Assuming that the student's knowledge does manifest itself only through the decisions he makes personally in appropriate situations, then the teacher cannot tell him what he does, nor determine his decisions, because in that case, he would give up the possibility the student to produce them, and also to "teach" them (Brousseau, 2008, p. 76).

From this moment, the first TSD dialectic moment begins, which is the action phase. On that occasion, the student, in the face of the problem, proposed by the teacher, will act on it, trying to develop strategies, intuitive or rational, in the face of the information included in the problem. The teacher, in turn, should encourage him to explore the problem and establish strategies based on the information contained in it.

Therefore, it is expected that he identifies, in the matter, the requirement of the concept of flat figures and measures of internal angles. Based on this, the student must deduce that the initial procedure will be to recognize the measure of the sum of the internal angles of an irregular pentagon (since it does not have all its sides equal). However, it is clear that, in the classroom, the study of the characteristics of the pentagon is generally not as thorough as that of the triangle. This type of obstacle, then, is characterized as a type of didactic obstacle, since, in general, the teacher prefers to explore, in his classes, the triangle (probably because it is more common to explore its specificities) to the pentagon (or another figure less notable). Therefore, it is assumed that, on this occasion, the student will have the initiative to divide the figure's polygon into triangles. From this moment on, the opportunity to explore and manipulate the dynamic resources in GeoGebra appears, and the teacher should encourage students to build the graphical representation of the polygon, divided into three triangles, for a better interpretation of the problem, as shown in the following figure 3:

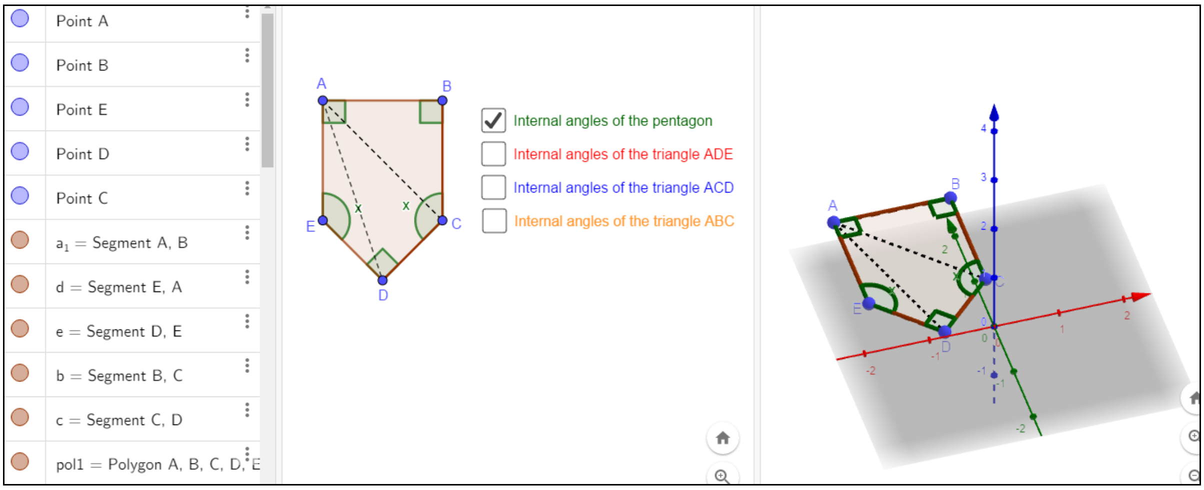

Figure 3. *Graphical representation in 2D and 3D constructed in GeoGebra of the ABCDE polygon decomposed in three triangles (Prepared by the authors.*

Next, Figure 4 is presented, which, in addition to being fragmented into three triangles, highlights the internal angles related to each triangle that makes up the pentagon:

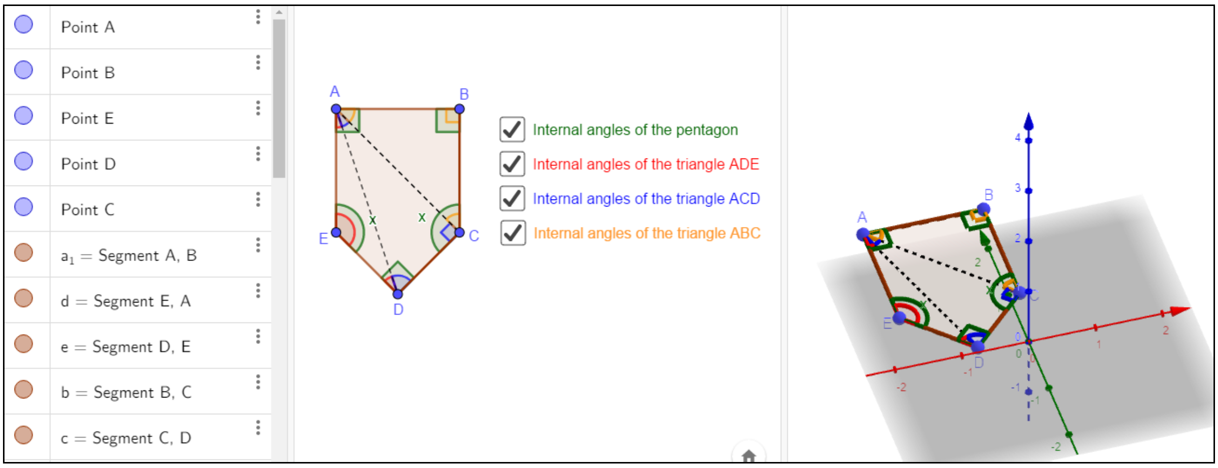

Figure 4. *Graphical representation in 2D and 3D constructed in GeoGebra of the ABCDE polygon decomposed in three triangles with their respective internal angles. (Prepared by the authors)*

In the formulation situation, according to Almouloud, “the student exchanges information with one or more people, who will be the senders and receivers, exchanging written or oral messages” (2007, p. 38). Therefore, the teacher must organize the students in groups, in order to induce them to a debate of opinions and strategies. The student, in turn, must already have some defined models or outlined theoretical schemes, even if the justification for these results has not yet been proven (ALVES, 2016). It is important to emphasize that during this moment of discussion, students must use a language that is understandable to everyone. Observing from Figure 5 that when drawing diagonals, from vertex A (or any other), in the ABCDE polygon, the triangles will form: ADE, ACD and ABC. Now, students are expected to realize, without delay, that the sum of the pentagon's internal angles will be equal to the sum of the internal angles of each of the three triangles, as shown below:

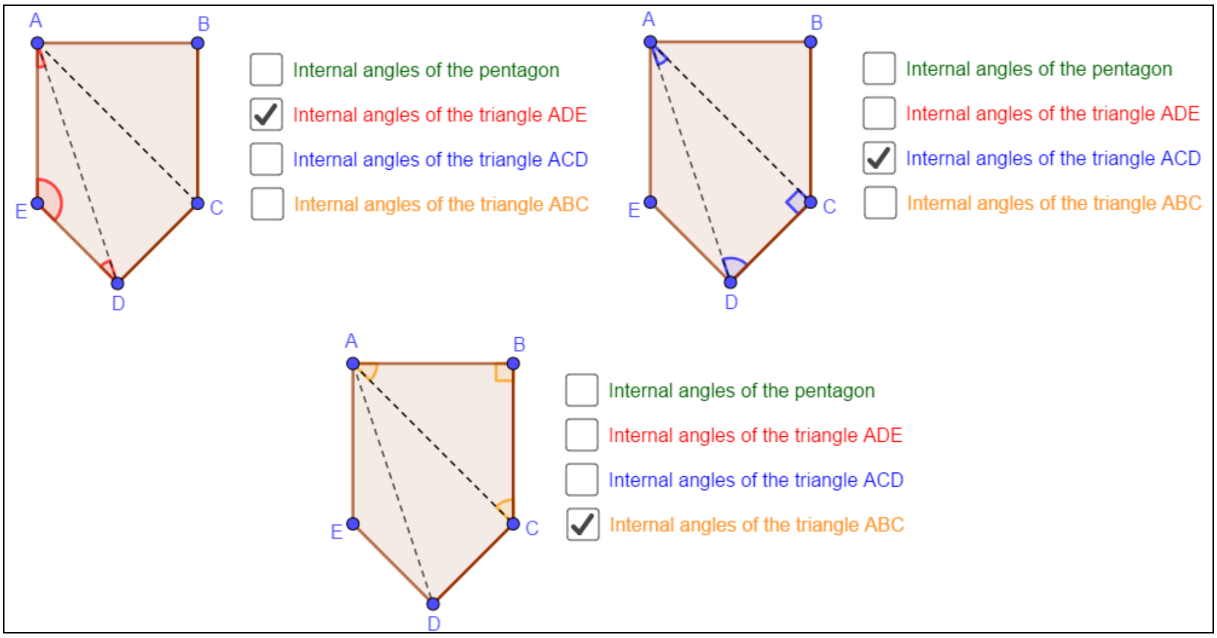

Figure 5. *Graphical representation constructed in GeoGebra of the ABCDE polygon decomposed in three triangles with their respective internal angles. (Prepared by the authors)*

Upon reaching this reasoning phase, visualization enters the scene once again, so that students can understand, analyze and compare the illustrative elements of the question, with those of GeoGebra, as shown in the following figure 6:

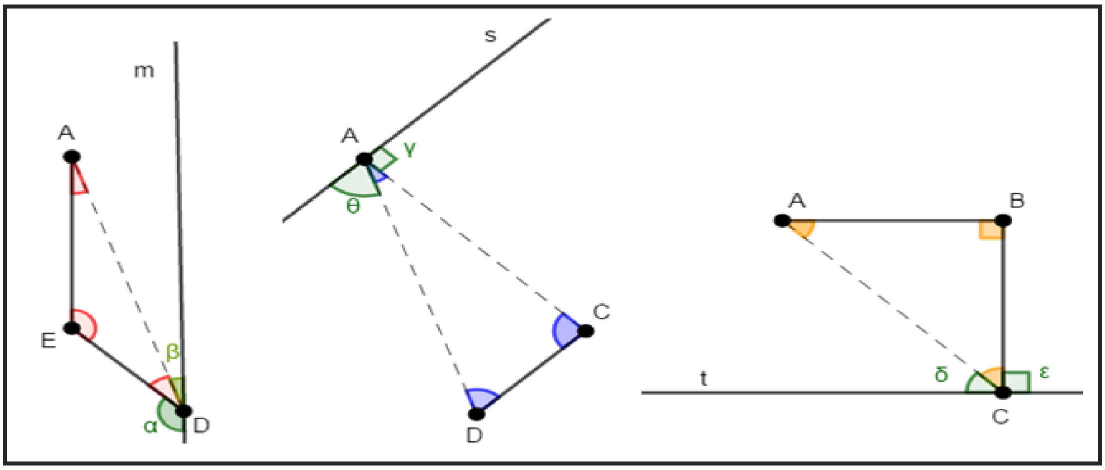

Figure 6. *Graphical representation of triangles ADE, ACD e ABC built by the software GeoGebra (Prepared by the authors)*

From Figure 6, when observing the triangle ADE, students should notice that the line m passes through point D and is parallel to the side $\overline{AE}$. In addition, the angles α, β and $\hat{D}$ they form a shallow angle and therefore measure together 180 °. Also knowing that the angles β and $\hat{A}$ are internal alternates and the same occurs with α and $\hat{E}$, they can come to the conclusion that $\beta = \hat{A}$ and $\alpha = \hat{E}$.

With that, they should note that the sum of the internal angles of the ADE triangle (and any other) is equal to 180º. It can be seen that the same occurs with the ACD and ABC triangles. With this prior knowledge, one can then proceed to the next phase.

In the validation phase, students must convince others about the veracity of their statements and assertions, requiring the use of an appropriate mathematical language (Teixeira; Passos, 2013). For that, it is necessary to use mechanisms of proof and demonstration, where it will be evident “the character of the truth and the elimination of possible inconsistencies and incongruities of the arguments used” (Alves, 2016). It is hoped, then, that the student explains that:

Sum of the pentagon's internal angles ABCDE = Sum of the internal angles of the three triangles, that is: $S_{ABCDE} = S_{ADE} + S_{ACD} + S_{ABC}$. Finally, through the ADE triangle in figure 6, we have that: α + β + $\hat{D}$ = 180º. As previously explained, β = $\hat{A}$ and α = $\hat{E}$, and the same occurs with the other triangles in the figure, we have that the sum of the internal angles of:

- triangle ADE: $\hat{A} + \hat{D} + \hat{E}$ = 180º;
- triangle ACD: $\hat{A} + \hat{C} + \hat{D}$ = 180º;
- triangle ABC: $\hat{A} + \hat{B} + \hat{C}$ = 180º.

Finally, S_{ABCDE} = 180º + 180º +180º = 540º.

Finally, the institutionalization phase is the moment when the teacher “conventionally fixes the cognitive status of knowledge” (Almouloud, 2007, p. 40). In this final dialectical phase, the teacher resumes the part of the responsibility, previously given to students at the time of return, resuming control of the situation, corroborating the true statements or discarding some of the students' productions (Teixeira; Passos, 2013).

Therefore, the teacher will be able to propose that students visualize the problem data with those produced with the GeoGebra software, so that they can confront them. Once this is done, the teacher will identify the singularities of each method used, review the concepts used, make the necessary corrections, in an attempt to make the content official, informing students of some proposals, taking into account everything that was produced and explored in the previous steps, such as:

1. The sum of the measures of the internal angles of any triangle is equal to 180º.

2. The sum of the internal angles of a pentagon is equivalent to 540º, as previously attested with the aid of Figure 6.

3. The formula for the sum of the internal angles of a polygon is given by the expression: S = (n - 2) · 180º, where n represents the number of sides of the polygon (this proof can be done for any polygon using the same strategy used to the pentagon in Figure 6).

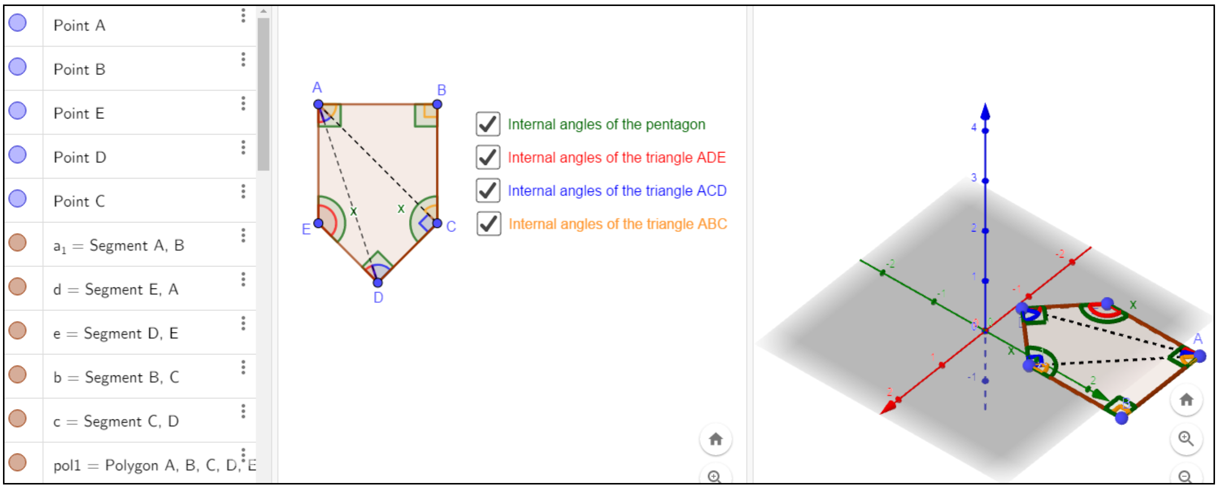

Figure 7. *The GeoGebra software allows the exploration of different viewing angles and mathematical investigation in a dynamic way. (Prepared by the authors)*

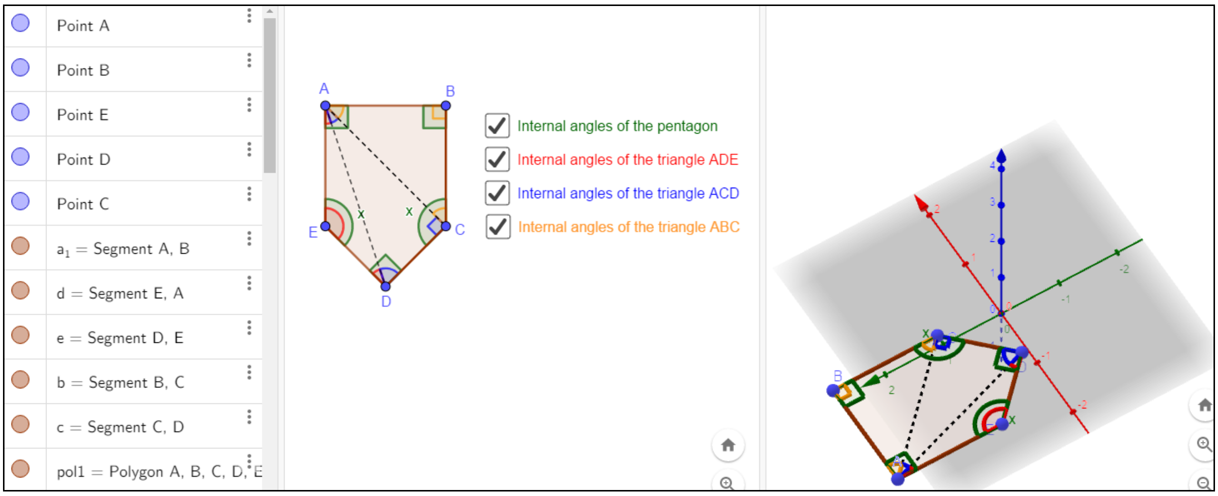

Figure 8. *The GeoGebra software allows the exploration of different viewing angles and mathematical investigation in a dynamic way. (Prepared by the authors)*

To conclude, in figures 7 and 8 we can see a corresponding construction with the didactic use of technology. The issue addressed in SPAECE Brazil, in a static and non-manipulative way, has a dynamic interpretation that stimulates the student's investigation and intuition. Similarly, such an approach requires more skills from the mathematics teacher. (Alves & Catarino, 2019).

5. Conclusion

This article represents introductory and theoretical data from an ongoing Master's research in the Master's program in Science and Mathematics Teaching in Brazil. At the time, we sought to show a proposal for a didactic situation related to Flat Geometry, aimed at SPAECE, in an attempt to provide the Mathematics teacher with a didactic resource to assist in the development of teaching content focused on this theme.

To this end, the Didactic Training Engineering (EDF) methodology was used, through its two initial phases (preliminary analysis and a priori analysis), enabling the construction of the didactic situation, which, while providing the organization of the research, also collaborates with teaching practice and assists in the development of their professional training and increase their skills in the use of technology.

At the time, we sought to consider the limitations, as well as the obstacles to the teaching and learning process, using some assumptions from the Theory of Didactic Situations (TSD), such as the didactic transposition and the didactic contract. Its dialectical phases, structured in the proposed didactic situation, aimed to guide the teacher to conduct pedagogical practices that would encourage the student to make decisions that would develop mathematical skills and, thus, make him the main character in the construction of his own knowledge.

The use of technological resources, through the GeoGebra software, proved to be a significant support in the absorption of concepts, by showing the visual elements and assisting in the management of the problem situation, giving the teacher the opportunity to provoke active involvement in the student, in a dynamic, representing the moment in a didactic transposition action, where visualization, perception and intuition are significantly present.

Finally, it is expected that this discussion and presentation of the proposal for a teaching situation will serve as input for teachers who intend to teach the concepts of Flat Geometry directed to SPAECE, aiming at the adoption of such methodologies in their teaching practices and the contribution to improving the teaching and learning process of mathematics.

References

Almeida, M. J. P. M. & Nardi, R. (2013). Relações entre pesquisa em ensino de Ciências e formação de professores: algumas representações. *Educação Pesquisa*, 39 (2), 335-349.

Almouloud, S. A. (2007). *Fundamentos da didática da matemática*. Paraná, Brasil: Editora UFPR.

Almouloud, S. A. & Silva, M. J. F. (2012). Engenharia didática: evolução e diversidade. *Revemat*, 7 (2), 22-52.

Alves, F. R. V. (2016). Teoria das Situações Didáticas (TSD): Sobre o ensino de pontos extremantes de funções com arrimo da tecnologia. *Revista Eletrônica Sala de Aula em Foco.* 5 (2), 59-68.

Alves, F. R. V. (2018). Engenharia Didática de Formação (EDF): sobre o ensino dos Números (Generalizados) de Catalan (NGC). *Educação Matemática Pesquisa.* 20 (2), 47-83.

Alves, F. R. V. (2019). A vertente francesa de estudos da didática profissional: implicações para a atividade do professor de matemática, *Vydia Educação*, 39(1), 255 – 275.

Alves, F. R. V. (2020). A didática profissional (DP): implicações para a formação do professor e o ensino de disciplinas específicas no Brasil. *Revista Iberoamericana de Estudos em Educação*, 15(4), 1903 – 1918.

Alves, F. R. V. (2021). Aperçu sur l'apprentissage et L'activite du professeur de mathematique: an point de vue derive de la Didactique Professionnelles (DP), *Acta Scientarum Education*, 43(1), 1 – 14.

Alves, F. R. V. & Catarino, P. M. M. (2017). Engenharia Didática de Formação (EDF): Repercussões para a formação do professor de matemática no Brasil**.** *Educação Matemática em Revista*, Rio Grande do Sul, 2 (18), 121-137.

Alves, F. R. V. & Catarino, P. M. M. (2019). Situação Didática Profissional: um exemplo de aplicação da Didática Profissional para a pesquisa objetivando a atividade do professor de Matemática no Brasil, *Indagatio Didactica*, 11(1), 103 – 129.

Artigue, M. (1988). Ingénierie Didactique. *Recherches en Didactique des Mathemátiques*, Paris, 9 (3), 281-308.

Artigue, M. (2014). Perspectives on design research: the case of didactical engineering. *Approaches to qualitative research in mathematics education*, 467-496.

Artigue, M., Haspekian, M. & Corblin-Lenfant, A. (2014). *Introduction to the Theory of Didactical Situations (TDS)*, In Networking of Theories as a Research Practice in Mathematics Education. Berlin: Springer, 47-65.

Bertagna, R. H., Mello, L. R. & Polato, A. (2014). Política e avaliação educacional: aproximações. *Revista Eletrônica de Educação*, 8 (2), 244-261.

Brousseau, G. (1976). Les obstacles épistémologiques et les problèmes en mathématiques. In J. Vanhamme & W. Vanhamme (Eds.), *La problématique et l'enseignement des mathématiques.* Comptes rendus de la XXVIIIe rencontre organisée par la Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques, Louvain la Neuve, 101-117.

Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics:* Didactique des Mathématiques, 1970-1990. Edição e Tradução de N. Balacheff, M. Cooper, R. Sutherland e V. Warfield. New York, Boston, Dordrecht, London, Moscow: Kluwer Academic Publishers.

Brousseau, G. (2008). *Introdução ao estudo da teoria das situações didáticas: Conteúdos e métodos de ensino*. São Paulo, Brasil: Editora Ática.

Chevallard, Y. (1991). *La transposición didáctica:* Del saber sabio al saber enseñado. Traduzido por Claudia Gilman. Buenos Aires, Argentina: Editora Aique.

Ceará. Secretaria da Educação do Estado do Ceará. SPAECE. (2013). *Boletim do professor – matemática*, Juiz de Fora/MG: CAED.

Ceará. Secretaria da Educação do Estado do Ceará. SPAECE. (2015). *Boletim do professor – matemática*, Juiz de Fora/MG: CAED.

Ceará. Secretaria da Educação do Estado do Ceará. SPAECE. (2019). *Boletim do professor – matemática*, Juiz de Fora/MG: CAED.

Pavanello, R. M. (1993). O abandono do ensino da geometria no Brasil: causas e consequências. *Revista Zetetiké,* Campinas, 1 (1), 7-17.

Perrin-Glorian, M. J. & Bellemain, P. M. B. (2019). L'ingenierie didactique entre recherche et ressource pour l'enseignement et la formation des maîtres. *Caminhos da Educação Matemática em Revista/Online*, 9 (1), 45-82.

Pommer, W. M. *A.* (2013). *Engenharia Didática em sala de aula: Elementos básicos e uma ilustração envolvendo as Equações Diofantinas Lineares***.** São Paulo, Brasil.

Ramírez, Bravo, R. (2005). Aproximación al concepto de transposición didáctica. *Folios*, Bogotá, Colômbia: UPN, 21, 33-45.

Teixeira, P. J. M. & Passos, C. C. M. (2013). Um pouco da teoria das situações didáticas (tsd) de Guy Brousseau. *Zetetiké*, 21 (39), 155-168.

Acknowledgments and financial support

The research development part in Brazil had the financial support of the National Council for Scientific and Technological Development - CNPq.

The research development part in Portugal is financed by National Funds through FCT - Foundation for Science and Technology. I. P, under the project UID / CED / 00194/2020.

Authors

Francisco Regis Vieira ALVES, Federal Institute of Science and Teconology of Ceara, Brazil, fregis@ifce.edu.br

Aline Maria da SILVA CAMILO, Federal Institute of Science and Teconology of Ceara, Brazil, allynecamilo@hotmail.com

Francisca Cláudia Fernandes FONTENELE, Universidade Vale do Acaraú, Sobral, Brazil. claudiafontenele05@gmail.com

Paula Maria Machado Cruz CATARINO, Universidade traz-os Montes e Alto D´ouro UTAD, Portugal. pcatarino23@gmail.com