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Modelling Difficulties and Their Overcoming Strategies in the Solution of a Modelling Problem

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Abstract: The purpose of the study is to reveal the elementary mathematics student teachers' difficulties encountered in the solution of a modelling problem, the strategies to overcome those difficulties and whether the strategies worked or not. Nineteen student teachers solved the modelling problem in their four or five-person groups, and the video records of their solution processes and their solution papers were examined. They had difficulties in simplifying, mathematizing, interpreting and validating and developed different overcoming strategies such as using the figure in the problem, real life experiences, utilizing a real object, answering the researcher, group discussion, sketching, reading the task back, considering calculation simplicity and some others peculiar to the problem. When they could overcome the strategies in simplifying and mathematizing, they could not be successful in tackling with those in interpreting and validating.

Key words: elementary mathematics student teacher, mathematical modelling, modelling difficulty, modelling problem, overcoming strategy.

1. Introduction

Students are taught mathematical concepts and methods in schools and they are able to use them only in solving mathematical problems. When they encounter daily issues out of the school, they have difficulty in using their mathematical knowledge to tackle with the issues. For students to be able to be part of social life, the real life applications to prepare them for life should be integrated into the lessons. Mathematical modelling, defined as transferring from a real life situation to a mathematical situation (Blum, 1993) and requiring real life experiences to reach meaningful solutions (Lesh & Caylor, 2007; Lesh & Doerr, 2003), has great importance to enable this integration.

The PISA 2006 results have revealed that students from all around the world have problems with modelling tasks and these problems were associated with the cognitive complexity of the tasks and derived from the necessity of using students' competencies (cited in Blum & Borromeo Ferri, 2009). Mathematical modelling is found difficult also by teachers because real world knowledge is required and teaching process becomes more open and less predictable (Blum & Borromeo Ferri, 2009). Crouch and Haines (2004) state the so-called difficulty is emerged from the interface between the real world problem and the mathematical model. Similarly Kaiser, Schwarz and Tiedemann (2010) indicate that modelling difficulties become particularly obvious in the area of real world and give it an example as the phase of simplification and idealization when the real world situation has to be interpreted and understood, and the real model has to be constructed.

Maaß (2006) prefers the term of mistake while explaining difficulties occurred during the modelling process. He explains the mistakes in setting up the real model arising from the assumptions simplified too much or distorted reality; the mistakes in setting up a mathematical model arising from not using adequate mathematical symbols and algorithms or formulas; the mistakes in solving mathematical questions within the mathematical model arising from wrong calculations, unfinished mathematical model or lacked heuristic strategies; the mistakes in interpreting the solution arising from missing the interpretation process or wrong interpretation; and the mistakes in validating the situation arising from missing validation, doing surface validation or not correcting the inadequacy of model (Maaß, 2006). Blum and Borromeo Ferri (2009) show some selected examples of students' difficulties and their reasons in different modelling task applications. They indicate students have difficulties in

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constructing models because of ignoring the problem context, just extracting numbers from text and calculate according to a familiar schema. Although students can construct an appropriate situation model, they cannot make assumptions in simplifying. When considering the difficulties in validating, they state validation is the most problematic phase and its reason is that students do not check whether solutions are reasonable and appropriate. Crouch and Haines (2007) explain that novice modellers can have difficulties in (i) holding the balance between the real world and the mathematical model, (ii) choosing an appropriate model to apply, (iii) deploying appropriate mathematical concepts and procedures to solve the problem, and (iv) linking the outcomes of the model back to the real world. It is stated that the novice modellers can have difficulty in understanding the problem and its context, and this situation can be effected by personal experiences with the context and also by the form of wording in word problems in another study of the researchers (Haines & Crouch, 2010). Blomhoj and Hojgaard Jensen (2003) state students have most difficulty in the sub processes named mathematization, mathematical analysis, and interpreting and evaluating the results.

The modelling difficulties are presented in various empirical studies and they are varied according to the student levels. Galbraith and Stillman (2006) considered modelling difficulties as blockages and constructed an emergent framework identifying student blockages in transitions between the modelling stages by using technology. They categorized the blockages in transitions from messy real world to real world problem, from real world to mathematical model, from mathematical model to mathematical solutions, from mathematical solutions to real world meaning of solutions and from real world meaning of solution to revise model or accept solution (Galbraith & Stillman, 2006). Schaap, Vos and Goedhart (2010) also considered the difficulties as blockages and identifies several modelling blockages and opportunities in terms of modelling process in a pilot study. In the phase of understanding, they explained the blockages in picking up the problem statement and its cause as impeding formulation of the problem text, overlooking essential parts in the problem text, or expecting hints, guidelines and necessary data in the problem text. The opportunities for this blockage were considered as describing the problem situation and exploring the problem. They observed a blockage in the phase of structuring as making erroneous assumptions and not recognizing a relevant variable, and explained its opportunity as drawing the problem situation and subconsciously simplifying the situation. In the phase of formalizing, not being able to convert specifications into a relation between variables and lacking algebraic skills were identified as blockages and scouting the problem using concrete examples, searching for a model by using an inductive method, and verifying the model by estimation were presented as their opportunities. Bukova Güzel (2011), in her study examining the process of pre service mathematics teachers' posing and solving modelling problems, indicated they had difficulties in interpreting and validating stages. Sol, Gimenez and Rosich (2011) researched 12-16 ages students' modelling behaviors and presented their difficulties as making connections between real objects and mathematical knowledge, identifying variables and correlating them and validating the model. When Hıdıroğlu, Tekin Dede, Kula and Bukova Güzel (2014) examined high school students' solution approaches on a modelling problem, they indicated the most problematic parts in modelling process were making assumptions and constructing models by using them, interpreting and validating. They stated the participants did not interpret or validate their solutions after reaching numerical solutions. In another study, identifying fourth year students' difficulties in the solution process of a model eliciting activity, the students experienced difficulties in understanding the problem, discovering the relations between the components of the qualitative variables, associating all of the variables with each other, hypothesizing, creating a suitable model based on their hypotheses, and validating the model by associating it with real life (Eraslan & Kant, 2015). In a PhD study aiming at the development of sixth year students' cognitive modelling competencies, Tekin Dede (2015) revealed the students who had no experience in modelling had difficulties in simplifying, mathematizing, working mathematically, interpreting and validating at the beginning of the study. They advanced in each stage of modelling process in consequence of implementations of action plans developed for overcoming those difficulties. Yılmaz and Tekin Dede (2016) investigated elementary mathematics student teachers' mathematization competencies and indicated they simplified data in a modelling problem more than adequate and had difficulty in validating because of not considering their real life experiences.

Even though the difficulties presented in the above-mentioned studies in the parallel of modelling stages, each are varied with the factors such as participants' class levels, performances, pre-knowledge, readiness and also implementation process, the content of tasks, appropriateness of task to participants and so on. All difficulties according to the phases of modelling process can be observed empirically and they are specific for tasks and students (Blum & Borromeo Ferri, 2009). On the other hand many studies reveal similar difficulties in similar stages of modelling process can be encountered although task and student levels differ (Biccard & Wessels, 2011; Blum, 2011; Blum & Leiß, 2007; Ji, 2012; Maaß, 2005; 2006; Peter Koop, 2004). There are various difficulties through the process and students have blockages are indispensable but the important point is here how to overcome these. Because the way to raise individuals successful in their real lives is to enable them to overcome the difficulties encountered. It is aimed to find out what difficulties the elementary mathematics student teachers confronted in the solution process of a modelling problem and what strategies they developed to overcome these difficulties in this study. Although the developed strategies are dealt with in the context of the given modelling problem, they present solution suggestions for the difficulties encountered frequently in the modelling process in general. In this regard, this study differs from the ones in the related literature because of taking into account their overcoming strategies along with the difficulties.

2. Method

The execution of modelling process with the student teachers were realized in this study while they solved a modelling problem. Since the difficulties they encountered and the strategies they developed to overcome those difficulties in the solution process were examined thoroughly, the case study research method (Cresswell, 2013) was utilized.

Participants. This study was conducted in a state university of Turkey with nineteen elementary mathematics student teachers in the Faculty of Education. The researcher met up with the elementary mathematics student teachers and made an explanation about conducting a study requiring mathematical modelling. She asked them to be participated in the study and would be informed about mathematical modelling as an additional course if they accepted participation. Since they had not encountered mathematical modelling during their education, nineteen of them accepted the participation voluntarily. So the criterion sampling technique was used in choosing the participants and the volunteering was considered as the sampling criterion. They were informed about the modelling applications and the modelling process by the researcher before the study. They were asked to form working groups in accordance with their preferences and so four working groups which are four or five-person worked on the modelling problem.

Data Collection. The data collection tools are the transcriptions of video records of the solution process and the papers on which their solutions are written. The groups solved the Fuel Problem (Tekin, 2012) at the same time in a class without any time limitations and each of them were videotaped synchronously. The participants were asked to construct a mathematical model about the rest fuel by considering the wet part of the stick sank in the tank (see Figure 1).

FUEL PROBLEM

The driver Ali suffers from a problem in the land trips and shares it with Mehmet who is an academician in the agricultural faculty. The problem is about his vehicle's fuel gauge. He says the fuel gauge is broken and he is not sure whether the fuel is enough to complete the trip or not. Because there is no fuel station on the road and he would be stuck in the middle of the land. Ali asks whether he can find out the fuel condition by using a very simple tool: "Mehmet, if I sink a stick into the fuel tank straight, can I learn how many litres fuel in the tank by considering the wet part of the stick? I request you to develop a model to be used to calculate how many litres left in the tank by measuring the wet part."

Figure 1. *Fuel Problem* (Tekin, 2012)*

* This problem was designed by the mathematics teachers after an in-service training on mathematical modelling.

A solution plan (see Appendix 1) was given to the groups with the problem and it included the stages of the Modelling Cycle under a Cognitive Perspective (Borromeo Ferri, 2006). They were asked to solve the problem in line with the stages of the cycle which were simplifying and mathematizing the problem, working mathematically, interpreting and validating the solution. The reason why the participants solve the problem according to the modelling stages is to reveal the difficulties transparently stage by stage in the course of the solution process.

Data Analysis. In the analysis of the data, coding process was realized in the frame of the stages of the so-called modelling cycle (Borromeo Ferri, 2006). The data were analyzed according to the codes previously determined (Cresswell, 2013) so the difficulties and how the participants overcame those were identified. A second coding was realized nearly one year after the first coding in line with the stability method (Krippendorff, 1980; Weber, 1985) and a percent agreement (Miles & Huberman, 1994) achieved greater than 70% between two coding. The direct statements of the participants were included in the results to present the concerned difficulty and its overcoming strategy.

3. Findings

The results concerning the difficulties, the overcoming strategies, whether these strategies work or not and the participants' expressions are presented in reference to the modelling stages are presented in the tables in this section.

The difficulties the participants encountered in simplifying stage and the strategies for overcoming them are presented in Table 1.

Table 1. *The Difficulties in Simplifying Stage and the Overcoming Strategies*

DIFFICULTIES IN SIMPLIFYING	STRATEGIES FOR OVERCOMING DIFFICULTIES			
	GROUP 1	GROUP 2	GROUP 3	GROUP 4
D1: Determining the shape and the location of the fuel tank	• Using the figure in the problem • Using real life experience • Utilizing a real object • Considering the fuel as a solid in right cylinder • Group discussion	• Using the figure in the problem • Using real life experience • Utilizing a real object • Group discussion	• Utilizing a real object • Sketching • Turning the paper • Group discussion	• Sketching • Imagining the tank perpendicularly • Group discussion
D2: Determining the shape of the base area	• Answering the researcher • Group discussion			

While simplifying the given problem, whole groups had difficulties in determining the shape and the location of the fuel tank (D1). They overcame this difficulty by developing strategies of using the figure in the problem, using real life experiences, considering the fuel as a solid in right cylinder, utilizing a real object, sketching, turning the paper, imagining the tank perpendicularly and discussing in their groups. When the participants in Group 1 did not decide how to calculate the volume of the cylinder horizontally, they discussed on considering the fuel as a solid in right cylinder and utilized the water bottle to imagine it as seen below.

Ozan: There is some water in the bottle, for example, if we turn the cylinder like that, it will be full this much [shows by using the plastic bottle]. If we turn it like that, can we calculate its height?

Melih: Do we consider it as a solid?

Burçak: I mean, yes. Because we think the liquid fill the lower part of the bottle. We section the cylinder, upstand it, there will be volume calculation in this circumstance. At last, it is a three-dimensional object.

Only one group had difficulty in determining the shape of the base area of the tank (D1) after deciding the shape and the location. Group members' statements are given below.

Burçak: The volume of all solids is the multiplication of the base area and the height. We should only calculate this half round. But is it a half round? It is not [shows the base of the figure they draw].

Ozan: It can be a round.

Burçak: It can be ellipse.

Ozan: It can be anything lower than round.

Burçak: I mean it could be a half ellipse or a half round? Any other ideas?

Melih: Could it be a half ellipse?

Beste: What is the base of a cylinder? Isn't it a round?

Ozan: Round.

Researcher: You cut some of them and then righted it. At last that section is the base of the figure you constructed last.

Ozan: Let's complete it as a round. What is its area? This is the radius of the round. There is this place minus the area of the triangle, isn't there?

After all groups simplified the problem in other words developed appropriate assumptions, they started to constructed mathematical models based on their simplifications. In the mathematizing stage, the participants' difficulties on constructing mathematical models and their overcoming strategies are given in Table 2.

Table 2. *The Difficulties in Mathematizing Stage and the Overcoming Strategies*

DIFFICULTIES IN MATHEMATIZING	STRATEGIES FOR OVERCOMING DIFFICULTIES			
	GROUP 1	GROUP 2	GROUP 3	GROUP 4
D3: The condition whether the stick's volume affects the model			• Ignoring • Group discussion	
D4: The condition whether a linear relation between the fuel amount and the wet part of the stick is			• Using the figure in the problem • Using real life experience • Group discussion	
D5: Expressing the variables in terms of each other		• Group discussion		

D6: Determining the variable number	• Answering the researcher • Modifying the model • Group discussion	• Answering the researcher • Reading the problem back • Group discussion	• Answering the researcher • Reading the problem back • Group discussion	• Sketching • Group discussion
D7: Requiring the modification of the model based on the fuel amount	• Group discussion		• Validating • Using numerical values • Answering the researcher • Group discussion	• Sketching • Group discussion

When the groups were constructing mathematical models, they had difficulties in considering the volume of the stick (D3), forming a linear relation between the variables (D4), expressing variables in terms of each other (D5), determining the variable number (D6) and requiring the modification of the model (D7). They developed different strategies to overcome them such as ignoring, using the figure in the problem or real life experience, answering the researcher, modifying the model, reading the problem back, sketching, validating, using numerical values and group discussion. All groups had difficulties in determining the variable number (D6) while constructing mathematical models. They missed that the problem required the model construction only depending on two variables –fuel volume and the wet part of the stick. Their initial models involved more than two variables such as two different angles (α and β), the wet part of the stick (x) and the radius (r) (see in Figure 2).

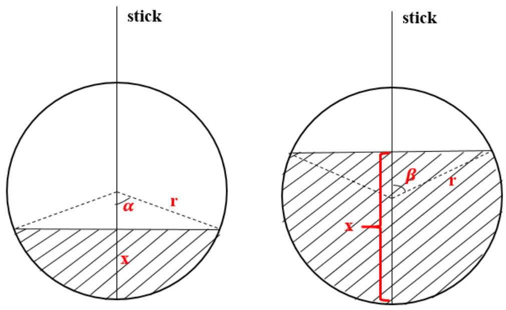


Figure 2. *The figure based on the initial models*

When the researcher asked question on the existence of variables or they read the problem back or sketched the fuel tank, they realized their mistakes and corrected it. For example, Group 2 read the problem back by means of the researchers' question and after a group discussion they could modify their mathematical model as follows:

Researcher: What is asked in the problem and what did you find? Do these meet each other?

Fadime: [Eda is reading the problem.] He is saying if I know the value of x, whether I can solve it or not.

Eda: Is there x in our equation?

Elif: Yes yes.

Fadime: Others are already standard. For instance he can measure the tank. x is the only unknown value here in my opinion. He can already measure the value of r.

Elif: There are α and β any longer. Also there is h.

Fadime: He can know them.

Elif: Wait, h is already known, it is height of the cylinder. Also r is known. Well what about α and β?

…

Elif: Values of α and β changes according to x.

Sümeyra: Sure, they depend on x.

Fadime: Let's represent α and β depending on x.

Sümeyra: For example something such as r.sin α instead of x.

Elif: No, we will write α and β depending on x. Then let's find α first.

Eda: Is cos α=(r-x)/r ?

Fatih: There will be r.sin α then.

Eda: arccos⟦((r-x)/r)⟧=α.

Elif: Here is r.cos α. If r-x=r cos α, we leave x alone.

Eda: Okay, α is this. Let's find β.

All groups except one had difficulty in modifying the model in case the fuel was more than half of the tank (D7). Group 4 drew a sketch of the fuel tank including two situation which were more and less than half of the tank and group members' expressions can be seen below.

Kezban: I want to say something, this model will change if the fuel is more than half of the tank. We will add it on the area of triangle.

Evrim: I think we will subtract it from the whole.

Kezban: Nothing will change. Angle α in the triangle will be another side in the equation then.

Evrim: Now we will subtract this from the whole.

Kerime: But α will be there now.

Elif: Alright then, we will form two different models.

Kerime: If there is α, it won't be a triangle. I wonder if the area of the triangle is negative. If α is more than 180, will it be negative? Can be?

Ayla: Now I understand what you said. But two of them could be the same.

Kerime: Ok then, the model becomes negative if α is more than 180. The main model doesn't change. This model is negative. We subtract this model. Because we subtract negative from negative, we actually obtain positive model.

All groups had difficulty in interpreting the solution in the context of real life and they discussed in their groups but they did not overcome this difficulty (see Table 3).

Table 3. *The Difficulties in Interpreting Stage and the Overcoming Strategies*

DIFFICULTIES IN INTERPRETING	**STRATEGIES FOR OVERCOMING DIFFICULTIES**			
	GROUP 1	**GROUP 2**	**GROUP 3**	**GROUP 4**
D8: Interpreting the solution in the context of real life	• Group discussion	• Group discussion	• Group discussion	• Group discussion

Group 2 did not consider reality while interpreting, they just stated what is asked in the problem and their performance to fulfil it. An excerpt from their expressions are as follows:

Elif: Let's interpret now. It asks us to find the length of x. We have already known r and h depending on variable x. Write down it. r and h which are the radius and the height are already known.

Sümeyra: r and h are already known. What did we do here?

Elif: We are trying the find the value of x.

Sümeyra: He has already known x too. He will write the values and find the volume. He will find the value of cosines depending on r and h. He will replace x into the model. Our model has already included r, h and x. He knows three of them and finds the volume directly by using them. That is it.

Similarly Group 4 expressed the interpretation as writing variables in terms of each other and their statements are given below.

Elif: Now we are writing what it is relative to what in interpreting.

Ayla: It is changing relative to α, h and t. α is increasing according to the amount of the fuel inside. As long as α increases, the volume increases.

After their inadequate approaches about interpretation, the groups tried to validate their solutions. While validating, they just preferred to consider numerical values and so had difficulty in corresponding numerical values taken into account (D9). They used real life experiences, spanned and estimated the dimensions, considered calculation simplicity, used figure in the problem, answered the researcher, expressed variables in terms of each other and discussed in groups to overcome this difficulty (Table 4). Despite they developed different overcoming strategies, they could not overcome the difficulty because of their insistence on calculation checking.

Table 4. *The Difficulties in Validating Stage and the Overcoming Strategies*

DIFFICULTIES IN VALIDATING	STRATEGIES FOR OVERCOMING DIFFICULTIES			
	GROUP 1	GROUP 2	GROUP 3	GROUP 4
D9: Corresponding the numerical values taken into account in validation	• Using real life experience • Spanning and estimating the dimensions • Considering calculation simplicity • Group discussion	• Using the figure in the problem • Using real life experience • Considering calculation simplicity • Group discussion	• Answering the researcher • Using real life experience • Using the figure in the problem • Considering calculation simplicity • Group discussion	• Using real life experience • Express variables in terms of each other • Group discussion

Group 1 used their real life experiences about the tanks, spanned the real objects and estimated the dimensions of the tank in the problem while corresponding numerical values to the variables in the model. In the meantime they gave priority to the calculation simplicity. This situation also removed their approach from the reality. Their expressions about these strategies are given below.

Melih: We need to find the values of h, r and a, don't we? What is h?

Ozan: As far as I see in vehicles, it should be like that.

Melih: [Measuring by the span the distance Ozan shows.] 75 cm.

Beste: How did you measure it?

Melih: One of my span is 20 cm. There are three spans and a little more. What is r?

Ozan: Take r 20 cm.

Melih: h is 70 now.

Burçak: Why? Do you want to simple calculations? Does it ask us α? We will assign a value to α then.

Ozan: Stop, stop! We should assign a value to α to enable not to find a complex solution.

Beste: Let's take a value enabling a perfect square.

Ozan: We should take a simple thing enabling to find the value of arcsines easily.

Melih: It doesn't matter. We will enter the value to the calculator and the machine will find already an approximate value. I will calculate the volume then. What is the value of π? 3 or 3,14?

Burçak: Take it 3.

Melih: If he use calculator, he should take it 3,14.

Ozan: In this case he enter 22/7 instead of π. So it can be more realistic.

Beste: He can simplify 70 by using 22/7... Take it 3!

Melih: No way! He will use a calculator.

Ozan: Why are we approaching it such professionally? It doesn't matter so much. He will take it 3.

Melih: Sure it will effect it.

Ozan: The result is 0,07 litre.

Melih: He has 0,07 litre fuel and he cannot go anywhere. He will be stranded!

Group 4 approached validation as checking the calculations and they expressed variables in terms of each other by using their real life experiences as follows:

Kerime: Isn't it a lorry? The fuel tank of a lorry?

Elif: It seems like a terrain vehicle.

Kerime: I think it is 1 m long. The tank is 1 m long. Look, it is like a huge lorry.

Evrim: There exists 50 cm.

Kerime: 50 cm is something like that [she shows it with her hands], 50 cm is small. Cars have such tanks scarcely. It should be 1 m.

…

Evrim: Let's examine the situation that the tank is full to validate it… Now we are going to validate it. We can find the situation the tank is full when we know the values of r and t indeed. If we take a value to h enabling to fill the tank completely, the two of them will be equal. Let's calculate it like that.

Kerime: We take the value of h as 2r then.

Evrim: After we find the lengths of r and t, we can find the volume of the tank. By multiplying the area and the height. I say we can find it and take it h as 2r and as 60 cm. We write whole values instead of the variables, will we reach the same result?

Elif: Let's do it! We will take π as 3, won't we? When h equals to 2r, we will take it 60 cm. [She is writing t=10 dm, r=3 dm, h=6 dm and π=3]

Elif: [She is making the calculation by writing the values into the variables.] Calculate the arccosine.

Zeynep: -1/2.

Elif: What is arccos(-1/2)?

Zeynep: 120.

Elif: It is180 dm3.

Zeynep: 180 lt.

Evrim: Now we are going to consider πr2 h.

Elif: This is now πr^2 t. 270 dm3.

Ayla: I don't understand what it should be. Should it be the same?

Kerime: It should be the same because we take it 2r.

Elif: Did I calculate wrong? Control it.

Evrim: It is -1. [She is correcting the arccos (-1/2) as arccos (-1).]

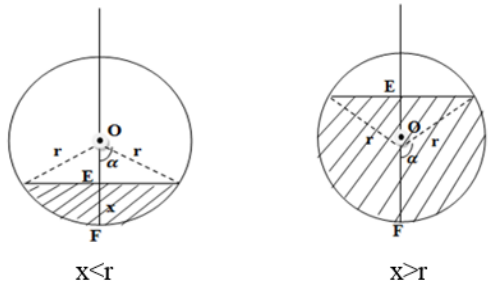
Elif: Done. [Correcting whole calculations.]

Evrim: If it is -1, there will be 180.

Kerime: It is true! What a relief!

Elif: We validated it in this case this model can be used.

Whole groups completed the solution process after they wrote their thoughts under the all titles in accordance with the modelling stages in the solution plan. Even though the groups had difficulties in the solution process, all of them were able to construct the mathematical model appropriately where x was the wet part of the stick, r was the radius of the base are of the tank and h was the height of the tank as in Figure 3.



$$V_{tank} = h\frac{r^2}{2}\left(\pi\frac{\cos^{-1}\left(\frac{r-x}{r}\right)}{180} - \frac{1}{2}\sin 2(\cos^{-1}\left(\frac{r-x}{r}\right))\right)$$

***Figure 3.** The mathematical model constructed in the problem*

4. Conclusion, Dicussion and Suggestions

In the study aiming at revealing the difficulties and their overcoming strategies of elementary mathematics student teachers, it was seen the difficulties were centered upon the simplifying, mathematizing, interpreting and validating stages. While the participants overcame the difficulties in simplifying and mathematizing, they could not dealt with those in interpreting and validating.

The difficulties in simplifying were derived from the deficiencies in the participants' real life experiences about the problem context and considering a mathematically simpler solution as making assumptions. Similarly Blum (2011) stated the students who had inadequate experiences about the reality of the problem had problems in simplifying it. Maaß (2006) and Lesh and Caylor (2007) expressed the students made richer assumptions if they met with the problem situations which were comprehensible in their reality. Also it was indicated that the students who tend to simpler solutions made redundant simplifications in modelling (Maaß, 2005; 2006). On the contrary, participants simplified the problem properly even if they considered simpler solutions. The reason why they simplified the problem with realistic assumptions were thought to use the tank figure given in the problem and draw a sketch correspondingly to the figure and the reality.

Mathematizing stage was identified as the one in which the participants had great difficulty. The problems were mostly originated from the variable variety and number. Despite the participants had more comprehensive mathematical knowledge, they were in tendency to use simpler models containing linear relations or ratio and proportion in the initial models. When they developed different strategies to overcome the difficulties in mathematizing, they realized they should construct mathematically valid models. In other words, their strategies could enhance their mathematical models. On the contrary, Biccard and Wessels (2011) and Blum (2011) expressed that the students constructed simpler mathematical models in initial modelling applications even though they had better mathematical knowledge. Schaap, Vos and Goedhart (2010) also stated the mathematical knowledge effected the constructed mathematical models. In this study, it was also thought the participant cumulated various and extensive mathematical knowledge throughout their undergraduate degree and this knowledge had great impact on constructing their models. Because all groups formed correct mathematical models depending on their assumptions and solved the models accurately, they did not have difficulty in working mathematically stage.

Since the participants knew they should interpret the solutions in the context of real life due to the stages written in the solution plan, they tried to interpret their solutions. When their group discussions were examined, it was understood they could not know how to interpret the mathematical results in a real context and so they did not overcome interpretation difficulties. This result indicated the modelling experiences of the participants came into play in interpreting. Similarly researchers stated the most problematic part of the modelling was interpretation phase and its most important reason was the lack of modelling experience (Biccard & Wessels, 2011; Blum, 2011; Bukova Güzel, 2011; Eraslan & Kant, 2015; Hıdıroğlu, Tekin Dede, Kula & Bukova Güzel, 2014; Ji, 2012; Maaß, 2006; Tekin Dede & Yılmaz, 2014; Peter Koop, 2004; Yılmaz & Tekin Dede, 2016).

When taking into account the participants' statements about validation, they firstly considered it replacement of numerical values into variables in mathematical models. However their practices about validation indicated they only considered the numerical values enabling the calculation simplicity and thus the validation went beyond the ordinary. In other words they did not overcome the difficulties in validating since they did not control their assumptions, mathematical models and solution of models. This situation had parallels with the other research's results which include the consideration of validating as only controlling calculation errors by the students had inadequate modelling experiences (Blum, 2011, Blum & Borromeo Ferri, 2009; Borromeo Ferri, 2006; Maaß, 2006).

Parallel to the most studies stating the modelling difficulties arose from the transitions from real world to mathematics and from mathematics to real world (Crouch & Haines, 2004; Kaiser, Schwarz & Tiedemann, 2010), it was remarked that the participants have difficulty in both transitions. However the essential factor reached in this study is that the participants could overcome the difficulties in transition from real world to mathematics although they could not overcome vice versa. The reason why they had difficulty in transferring mathematical results to the real world was thought as lacking of modelling experience and so being in tendency to complete the solution process after reaching a mathematical solution.

When considering the results of the study as a whole, interpreting and validating which are most problematic stages of the modelling cycle are suggested to emphasize on insistently during the modelling applications. Possible strategies to overcome the difficulties in these stages can be identified through different modelling applications with the students in different levels and so future modelling applications can be developed by taking into consideration them. Therefore it is thought to be provided more successful individuals in modelling.

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Appendix 1

SOLUTION PLAN

Simplifying the problem:

Mathematizing the problem:

Working mathematically:

Interpreting the solution:

Validating the solution: