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PRIMARY SCHOOL PUPILS' PROBLEM SOLVING COMPETENCY AND REASONING SKILLS

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Abstract: Developing problem solving competency is one of the most important goal of Mathematics teaching. The aim of this paper is to study primary school pupils' problem solving and reasoning skills. Pupils had to solve two logical problems (formulated as multiple-choice items) and they had to write down their argumentation. A quantitative (calculating the percentage of pupils selecting each choice) and a qualitative (analyzing pupils argumentations) analysis was done. The results show that pupils can't write an acceptable argumentation, their logical reasoning is based mainly on guessing. The research underline the dangers of using multiple-choice items for evaluating problem solving skills: pupils could select the correct answer without solving correctly the problem.

Key words: problem solving competency, reasoning skills, Mathematics teaching, primary school Mathematics

1. Introduction

Developing problem solving competency is one of the most important goal of Mathematics teaching. To develop their problem solving competency pupils need to solve non-routine problems. Solving these kind of problems pupils need to find new methods, combine the known methods and algorithms, use their knowledge in a new way, so they have to be creative. When solving a problem explaining the solution is important, because this shows that the pupil has understood the method. Thus, pupils' reasoning skills have to be also developed.

The aim of this paper is to present the results of a research made among 3rd and 4th grade pupils related with their problem solving competence and reasoning skills. A qualitative analysis on pupils reasoning is made, underlying the main mistakes they make when explaining the solution of a logical problem.

2. Theoretical background

A problem occurs when someone has a goal, but she/he does not know the way to achieve it (Duncker, 1945). "An individual is faced with a problem when he encounters a question he cannot answer or a situation he is unable to resolve using the knowledge immediately available to him." (Kantowski, 1977, p. 163) In case of a problem, the solver does not have an algorithm or method, which she/he could apply in order to lead to a solution (Kantowski, 1977). To be more specific, in many cases instead the "problem" the expression "non-routine problem" is used, to emphasizes the fact, that for solving a problem the algorithm or method is not known. "Non-routine problem" is also used in TIMSS 2011, where "non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned." (Mullis et al, 2009, p. 45).

Problem solving is a cognitive process, which transforms a given situation into a goal situation when the solving method is not obvious (Mayer, 1990).

Problem solving competency is "an individual's capacity to use cognitive processes to confront and resolve real, cross - disciplinary situations where the solution path is not immediately obvious and

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where the literacy domains or curricula areas that might be applicable are not within a single domain of mathematics, science or reading." (OECD, 2003, p. 156) Creative thinking and critical thinking are important components of problem solving competency (Mayer, 1992).

During problem solving some barriers could appear. These barriers could be related with lack of knowledge about the problem situation, lack of methods needed for solving the problem, etc. Overcoming these barriers needs not only cognitive skills, but also motivational and affective factors (Funke, 2010). Thus in PISA 2012 the definition of problem solving competency includes affective components too, i.e. problem solving competency is "an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen." (OECD, 2013, p. 4).

Mathematical problem solving needs application of multiple skills (De Corte, Verschaffel, & Op't Eynde, 2000). Schoenfeld (1985) identified four categories of knowledge/skills needed to be successful in mathematics: mathematical notions, problem solving strategies and techniques, decision abilities and beliefs about mathematics. Observe that Schoenfeld (1985) has included beliefs in the knowledge/skills needed for problem solving. To measure the cognitive processes essential for problem solving, it is better to avoid the need of domain specific knowledge and strategies (OECD, 2013). Logical reasoning skills are important for a successful mathematical learning (Nunes et al., 2007).

Pólya (1945) has identified four main stages of problem solving: understanding the problem, making a plan, carrying out the plan, and reviewing the solution. In mathematics education literature similar steps are described by many researchers, as Higgins (1997); Leader and Middleton (2004); Ridlon (2004).

3. Research

Research design

The goal of the research is to study primary school pupils' problem solving competence and reasoning skills. It is a qualitative research.

The research was made during February-March 2013. In the research 146 pupils have participated, 61 3rd grade and 85 4th grade pupils.

A problem sheet with two problems constitutes the research tool. These two problems were selected from problems given at Zrínyi Ilona Mathematical Competition. The problems are logical ones, no mathematical notions or methods are needed for solving them. Thus these problems can really test pupils' problem solving competence. In the competition problems are formulated in the form of a multiple choice item. In this research we asked additionally to write down the reasoning which led to the selected solution. The reason for choosing problems from this competition is the following: on the first level of this competition many pupils participate and the percentages of those choosing each option are published. Thus we could compare the results obtained in this research with the results of the pupils on this competition.

In Problem 1 there are given 4 sentences on an imaginary language. Then the pupils have to select the translated version of a sentence from five given possibility. When solving the problem, from (2) and (4) we conclude that I like = memme. Thus choice (C) and (E) is not correct, don't' contain the word "memme". From (1) and (3) we can conclude that fell down = bam; croissant = ham; from (3) that tree = fam. Thus the translation should contain "fam bam" (so (D) is not correct) and it shouldn't contain the word "ham (so (B) is not correct). In conclusion, the correct answer is (A).

This problem was given at the local phase of this competition. 14% of 3rd grade pupils and 24% of 4th grade pupils have chosen the correct answer (see Table 2).

Problem 1 (Csepcsányi et al., 2002).

In Tingling country the official language is anka. There are given four sentences in anka language:

(1) The croissant fell down. = Ham bam.
(2) I like soup. = Vele memme.
(3) From the tree fell down. = Bam fam.
(4) I like cacao. = Dudu memme.

What could be the translation of the sentence: I like to pick up the apples fell down from the tree?

(A) Memme venne fam bam ma.
(B) Memme ham fam bam ma.
(C) Vele dudu fam bam ham.
(D) Memme venne ham bam ma.
(E) Dudu venne bam fam al.

Problem 2 could be solved with logical reasoning. We could start, for example, from the affirmation of Dezső "Ernő is from Győr" and from the affirmation of Ernő "I am really from Győr". If we assume, that Ernő is from Győr, then these affirmations are true, so the second affirmation of Dezső and Ernő is false, so Dezső is not from Kecskemét and Anna is not from Miskolc. So Anna's affirmation that „Dezső is from Kecskemét" is false, thus her second affirmation is true, so we conclude that Anna is from Szeged. We got the answer for the question of the problem, but in order to be sure that our assumption from the begining (that Ernő is from Győr) doesn't lead to a contradiction, we need to continue our argumentation. Then Béla's affirmation, that he is from Szeged, is false, so Csilla is from Pécs. Similarly, Csilla's affirmation, that she lives in Szeged, is false, so Dezső is from Miskolc. So we didn't get any contradiction.

Problem 2 (Csordás et al., 2006).

At the Zrínyi Ilona Mathematical Competetion five participants living in five different cities (Győr, Kecskemét, Miskolc, Pécs és Szeged) affirm the following:

Anna: Dezső is from Kecskemét. I am from Szeged.
Béla: I am from Szeged. Csilla is from Pécs.
Csilla: I live in Szeged. Dezső is from Miskolc.
Dezső: I am from Kecskemét. Ernő is from Győr.
Ernő: I am really from Győr. Anna is from Miskolc.

Which participant lives in Szeged, if every child had a true and a false affirmation?

(A) Anna (B) Béla (C) Csilla (D) Dezső (E) Ernő

Table 1. Organizing the data from Problem 2

	Anna	**Béla**	**Csilla**	**Dezső**	**Ernő**
Anna	Szeged			Kecskemét	
Béla		Szeged	Pécs		
Csilla			Szeged	Miskolc	
Dezső				~~Kecskemét~~	**Győr**
Ernő	~~Miskolc~~				**Győr**

In case of this type of problems a table helps us to make the reasoning process more transparent (Table 1). In each row we mark the affirmation of the child from that row, i.e. in Anna's row we write „Szeged" for Anna and „Kecskemét" for Dezső. Then we cross out those affirmations, which considered to be false and highlight those ones, which considered to be true. So, if we assume, that Ernő is from Győr, then these affirmations are true, so the second affirmation of Dezső and Ernő is false.

This problem was given at the finals of this competition. 41% of 3rd grade pupils and 53% of 4th grade pupils have chosen the correct answer (see Table 2).

Table 2. Pupils choices at Zrínyi Ilona competition for the two problems

	3rd grade		4th grade	
	1st problem (%)	2nd problem (%)	1st problem (%)	2nd problem (%)
A	14	41	24	53
B	6	10	10	14
C	33	16	30	12
D	5	0	4	1
E	26	1	22	3
empty	16	32	10	17

Results and discussion

Table 3 contains the percentages of those choosing answer (A), (B), (C), (D) and (E). These results are presented separately for 3rd grade and 4th grade, and for all research participants.

Table 3. Pupils choices in this research for the two problems

	3rd grade		4th grade		Total	
	1st problem (%)	2nd problem (%)	1st problem (%)	2nd problem (%)	1st problem (%)	2nd problem (%)
A	14.75	24.59	15.29	18.82	15.07	21.23
B	22.95	45.9	21.18	25.88	21.92	34.25
C	22.95	14.75	24.71	47.06	23.97	33.56
D	4.92	0	2.35	4.71	3.42	2.74
E	27.87	6.56	25.88	2.35	26.71	4.11
empty	6.56	8.2	10.59	1.18	8.9	4.11

Analyzing problem 1

We could observe that 14.75% of 3rd grade and 15.29% of 4th grade pupils have chosen the correct answer. Comparing these results obtained by the participants of the Zrínyi Ilona Mathematics competition, we could observe that the results of 3rd grade pupils is similar with the results of the competitors (14.75% v. 14%), but 4th grade pupils perform less than pupils at the competition (15.29% v. 24%).

The less selected choice is (D), only 3.42% of pupils considered this to be the correct answer. This choice doesn't contain the word "fam", so pupils have found the translation tree = fam. 23.97% of pupils have selected variant (C) and 26.71% (E). These pupils didn't find correctly the correspondence of "I like", they thought that I like = vele respectively that I like = dudu. They didn't observed carefully propositions (2) and (4), they considered that the order of words are the same in the two languages. 21.92% of pupils have chosen the answer (B). These pupils have translated correctly the word "I like", but they didn't observe, that this proposition contains the word "ham", which means croissant.

Analyzing problem 2

We could observe that 24.59% of 3rd grade and 18.82% of 4th grade pupils have chosen the correct answer. These results are much lower than of the participants of the Zrínyi Ilona Mathematics competition (24.59% v. 41% for 3rd grade and 18.82% v. 53% for 4th grade). This difference is explainable by the fact that this problem was given at the finals of this competition.

It is interested that the percentages of those have chosen (B) respectively (C) is very different for grade 3 and grade 4. In case of variant (B), 45.9% of 3rd grade and 25.88% of 4th grade pupils have chosen it. In case of variant (C), 14.75% of 3rd grade and 47.06% of 4th grade pupils have chosen it.

We could also observe that only those children, who tell that they are from Szeged are frequent selected by the pupils, only 2.74% of the pupils have selected Dezső and 4.11% selected Ernő.

Argumentations for *Anna* being from Szeged:

"Anna, because she tells, that she is from Szeged." – We could observe, that even those children who have selected the correct answer didn't solve correctly the problem. Because Anna is the first child telling that she is from Szeged, pupils has chosen her as correct answer.

"Anna is from Szeged, the other children were lying."; "Anna tells the true, Ernő is lying" – Again these pupils have got the correct answer without correct argumentation.

„Anna is from Szeged, because for the other children after the „:" mark is strating with another word, except Béla." – This is an illogical answer, this child couldn't solve the problem, but if the problem is formulated as a multiple choice problem, she has selected the correct answer.

"Because Béla tells, that Ernő is from Győr, and this is true, his first affirmation is false, thus Csilla is from Pécs. Because the second affirmation of Dezső is true, then his first affirmation is false, thus Dezső is from Miskolc. The first affirmation of Anna is false, and the second affirmation is true." – This looks like a correct argumentation, but if we compare with the affirmations from the problem, we see, that Béla doesn't tell, that Ernő is from Győr.

„I thought, that the first affirmation of each child is a lie." – It is just a assumption, it doesn't base on any logical reasoning.

„I have seen that Ernő tells that Anna is from Msikolc, thus it turns out that Anna tells a lie." – This child have found two contradictory affirmations: Anna says that she is from Szeged; Ernő says that Anna is from Miskolc. She concluded that Ernő tells the true, she didn't considered the situation when Anna tells the true or both of them lie. It is interested, that this child concluded that Anna lied, but she selected Anna as a solution.

„I started with Ernő. Then I continued. In this way I arrived to the fact that Anna lives at Szeged." (see Figure 1) – This child has noted on the affirmations which one is true and which one is false. She worked correctly.

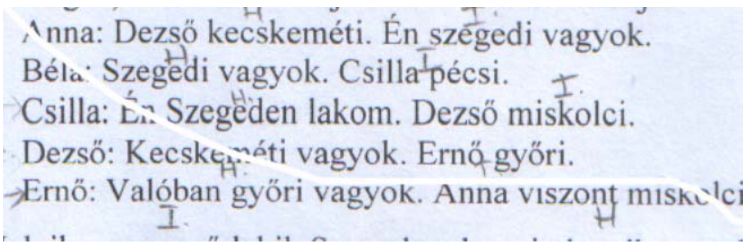
Anna: Dezső kecskeméti. Én szegedi vagyok.
Béla: Szegedi vagyok. Csilla pécsi.
Csilla: Én Szegeden lakom. Dezső miskolci.
Dezső: Kecskeméti vagyok. Ernő győri.
Ernő: Valóban győri vagyok. Anna viszont miskolci

Figure 1. Representation of a pupil: I (true) and H (false) marks near each affimation

Another representation used by some of the pupils is cutting out the false affirmations in case of each child (see Figure 2). He also worked correctly.

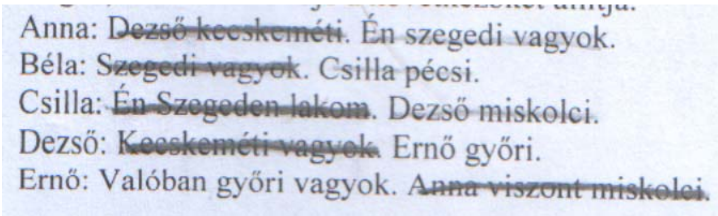
Anna: ~~Dezső kecskeméti~~. Én szegedi vagyok.
Béla: ~~Szegedi vagyok~~. Csilla pécsi.
Csilla: ~~Én Szegeden lakom~~. Dezső miskolci.
Dezső: ~~Kecskeméti vagyok~~. Ernő győri.
Ernő: Valóban győri vagyok. ~~Anna viszont miskolci~~.

Figure 2. Representation of a pupil: cutting out the false affirmations

We could observe that many children have selected Anna to be from Szeged, but they had an incorrect argumentation for their choice, so their solution was wrong. Anna is the first child telling that she is from Szeged, so some pupils have chosen her from this reason. We could see the danger of using multiple-choice items for evaluating problem solving skills: pupils could choose the correct answer without solving correctly the problem.

Argumentations for *Béla* being from Szeged:

"Anna is from Szeged, Béla is from Szeged, Csilla is living in Szeged. But Béla tells the true, because Anna is from Miskolc and Csilla is from Pécs." – This pupil has found all the sentences telling that someone is from Szeged, then he searched for affirmations about Anna, Béla and Csilla. He found affirmations about Anna and Csilla, telling that these girls are from other cities, so he has concluded, that Béla is from Szeged.

"Because nobody tells where Béla is from."; "Because nobody tells, that Béla is not from Szeged."; "Béla said that he is from Szeged. Nobody contradicts him." – These argumentations are not valid.

"Béla, because Anna for sure doesn't tell the true, as Ernő admitted that he is from Győr; Csilla for sure lies, Dezső for sure doesn't lie; Ernő admitted that he is from Győr." – In this arguentation the child missed up something, as Ernő didn't tell that he is from Győr.

Argumentations for *Csilla* being from Szeged:

"Csilla, because she tells that she lives at Szeged." – It is interesting, that pupils highlight the difference between *being from Szeged* and *living in Szeged.*

"Anna and Dezső tells the same. Because Anna's affirmation related with Szeged is a lie, the second affirmation of Ernő is true. Ernő is not from Győr, because his second affirmation is true, not the first one. If Ernő's first affirmation is not true, then Dezső's second affirmation is also a lie." – It is a good observation that Anna and Dezső tells the same. But the fact that Anna's affirmation related with Szeged is a lie is only an assumption and from this assumption doesn't follow that Anna is from Miskolc.

Some pupils tried some graphical organizer by writing near each name those places which appear in the affirmations (see Figure 3).

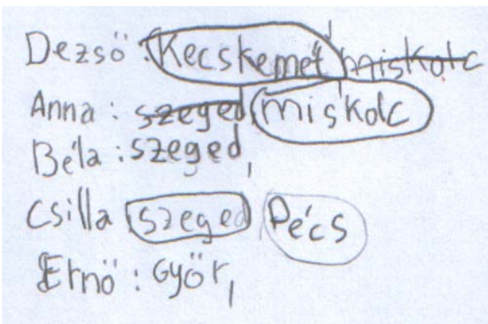

Figure 3. Representation of a pupil: writing near each name the places which appear in the affirmations

Argumentations for *Dezső* being from Szeged:

"Because his name is present already in the first line." – This is not a valid argumentation.

4. Conclusions

It is difficult for pupils to write down an argumentation for their solution, they can't make a logical reasoning. In many cases their argumentation is based on guessing. When pupils have to find the answers for a question by concluding it from more affirmations, in many cases they don't consider all the affirmations, they just make their conclusions based of some of arbitrary chosen sentences.

Problems formulated as multiple-choice items not always reflect real results, as the pupil could select the correct answer without solving correctly the problem. We could observe this phenomenon especially for problem 2.

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