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TEACHING A COMPUTER SCIENCE RELATED TOPIC IN PRIMARY SCHOOL

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Abstract: In the following article I share my experience with the teaching of a computer science related topic of binary number system to third grade primary school pupils. A story-problem published in the mid-thirties of XX c. exemplifies the necessity of using binary number system in a specific situation at the market. To demonstrate the idea of representing natural numbers by powers of 2, in the course of work I have used lentil grains as manipulatives. Easy to count and group, they created a unique classroom atmosphere which reminded the lifestyle of the time when the story-problem had been written. The nine-year-old schoolchildren were deeply intrigued by the topic and took the obstacles of writing numbers' binary codes as opportunities to make their own discoveries and achievements.

Key words: Binary Number System, Story-problems, Primary School, Manipulatives

Introduction

In the mid-thirties of the XX c. a children's newspaper named *Path*[1] circulated around Bulgaria. Its subscribers eagerly expected it not only because of the latest novel sequel or fairy tale. They found there puzzling story-problems and the whole family were likely to gather and look for the answers. The situations were represented through the words and actions of a middle aged man named Uncle Stanyo who was passionate about applying mathematical knowledge in everyday life.

Almost a century after it had been written, one of these problems grabbed the attention of a class of modern nine-year-old pupils. In order to be solved the concept of binary number representation had to be applied. It turned out that decades before the modern digital devices became a part of our life, the *Path* publishers had considered necessary that Bulgarian chidren from towns big and small knew why binary number system was needed and how it worked in practice.

1. The problem before the concept

Representing the numbers through the sequences of zeros and ones is the fundamental of storing and processing information in computers. As Schneider and Gersting (2016, p. 176) underline the main reason for that is related to electronic devices reliability. The importance of binary number system explains its place among the first topics which computer scientists study (e.g. Petzold, 2000, pp. 61−85; Schneider and Gersting, 2016, pp. 152−217; etc.). To introduce its idea to the third graders from 119th school in Sofia with whom I worked on a mathematical project, I started with the *Path* story-problem mentioned. Because of the limited class time, I slightly shortened it, but kept the characters, the dialogue style, and the spirit of the time. Here is my version:

Once a week Uncle Stanyo goes to the store to buy what his family needs together with some goodies for the neighbors' children. As small change is always a problem, today he wants to be prepared for his purchase and give the exact amount of money. His neighbors' children crowd around him and Uncle Stanyo decides to challenge them mathematically. He has prepared 1,000 levs[2] *to spend and asks his wife to bring the money and 10 empty purses. As soon as she does that, he says:*

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”Here, children, distribute these 1,000 levs, all in coins of one, into the ten empty purses in such a way that whatever sum I need to pay, I can use the purses without counting the coins in them. To make this easier for me, on each purse put a label how many levs it contains.”

How did the children label the purses? (See Elin Pelin and Podvarzachov, 1992, pp. 53–54)

While I was reading the text, the class listened deeply absorbed. It was not surprising because their textbook story problems are usually “stripped of their engaging details” (Zazkis and Liljedahl, 2009, p. 67). The third graders who regularly saw their parents make payments by credit and debit cards, were impressed by the detail that in these times going to the market had been associated with waiting for change. Thus beneath the surface of the vivid life story, quite naturally a serious mathematical task emerged.

Reading this problem, the professional mathematicians and computer scientists immediately recognize the idea of binary number system representation. But the *Path* publishers who were among the most prominent Bulgarian writers, neither had expert knowledge in the field nor required such from their readers. The purpose of the story-problem was to tell the children that a curious universal way for making payments existed; so, in order to find it, they had to rely on their inventiveness, creativity, and logical thinking. The concept of binary number system was not explicitly stated in the problem formulation. Yet the sentence “*Here, children, distribute these 1,000 levs all in coins of one into the ten empty purses*” could be interpreted as an unintrusive hint to explore the situation through error and trial, presumably using any small objects they had at hand instead of coins.

The engaging story made several pupils impulsively search in their pockets for 1-lev coins: the story-problem had attracted their attention to the situation. At this moment I put on each desk a plastic cup with lentil grains to be used as manipulatives. They were equal in shape, size, and weight just like the coins brought by Uncle Stanyo’s wife, a little bit flat and clean of detail. Therefore they were easy enough to count and group (*Figure 1*) for the purposes of the upcoming activities. Except for “building explicit bridges between the informal understanding and the formal symbolic instantiation” (McNeil and Jarvin, 2007) of the idea of binary number system, the lentil grains created a unique atmosphere in the classroom. It was close to the lifestyle in the mid-thirties of the last century and inspired the children to identify themselves with the story-problem characters. In such a way these manipulatives “reinforced the real-world scenario” (McNeil et al., 2009) of the story-problem.

Figure 1. *Using the lentil grains, the pupils made physical models of the first several powers of 2*

2. Assembling the building blocks of binary number system

Using their lentil grains as coins, the third grade students started distrubuting them into piles to model the purses filling. Their first arrangement consisted of 1 coin in the first pile, 2 coins in the second, 3 in the third, etc. Following this pattern, they placed 10 coins in the tenth purse just to complete the process and convince themselves that it did not work: all ten purses wefe full, no more than 55 levs could be paid by them, and there were still 945 levs left behind. Although not happy about the situation, the students assessed it as real professionals, concluding that “they did not effectively distribute the 1,000 coins and needed another pattern”.

During the last decades, “a new definition of mathematics as the *science of patterns*” emerged (Devlin, 2012, p. 3). As the students began talking about patterns, I pointed out that the well-known decimal number system was a pattern of number representation and other representations of the sort existed. Even if it might seem overrated, I formulated the Basis Representation Theorem (Andrews, 1994, pp. 8−10; Eves, 1983, p. 9) to the third graders without a proof, but providing enough explanations:

Theorem. (*Basis Representation Theorem*): *Let k be any integer larger than 1. Then, for each positive integer N, there exists a representation*

$$N = a_0 k^s + a_1 k^{s-1} + ... + a_s$$

where $a_0 \neq 0$ *and where each* a_i *is nonnegative and less than k. Furtherore, this representation of N is unique; it is called the representation of N to the base k.*

During the break, several pupils spontaneously repeated out loud the theorem for some particular bases, using their own words and asking me if they had formulated it correctly.

According to the Bulgarian math curriculum the exponents are not studied before the middle school. Therefore we had to discuss the building blocks of the decimal number system not as the powers of 10, but as the set {1, 10, 100, 1,000, …} and for the binary one – not as the powers of 2, but as the set {1, 2, 4, 8, 16, …}. We also talked about the digits “allowed” to write the numbers in positional number systems. The students knew that in the decimal system ten digits were used, but made a wrong analogy between it and the binary number system. They suggested using the digits of 1 and 2 instead of 0 and 1. Their curiosity grew after I shared some details about the hexagonal number system whose sixteen digits comprised of the ten usual digits from 0 to 9 completed with the first six letters of the English alphabet.

Entusiasted by the whole new knowledge acquired, the pupils promptly distributed the lentil grains (“coins”) in piles (“purses”) of 1, 2, 4, 8, … A pair of students put their piles on small pieces of paper with the number of grains written on them, thus labeling their first several purses (*Figure 2*):

Figure 2. *A pair of students labeled their piles of grains as the story-problem question required*

But the main part of the problem solving task was still ahead. Since the Basis Representation Theorem was not going to be proved, the pupils were to be sure that this was the right coin distribution and they were to be able to make the payments themselves.

To help the students select the necessary purses, I suggested the following approach: compare their weights with the weight of the amount of money to be paid in one-1ev coins. According to the story-problem, all 1,000 coins available were equal in weight, therefore the more money in a purse, the heavier it was. To pay, for example, 90 levs, the students were first to use the heaviest purse whose content did not exceed that money: it was the one containing 64 levs. For the 26 levs left, they had to apply the same rule: to take the heaviest purse whose content did not exceed that residual, i.e. the one with 16 levs. Then for the rest 10 levs, the purse with 8 levs was to be chosen. For the last 2 levs left, taking the purse with 2 levs finalized the process. In such a way, the pupils represented number 90 as a sum of the following powers of 2: 90 = 64 + 16 + 8 + 2 and continued with several more examples. *Figure 3* shows not only one student's neat writing, but also his good understanding of the approach I explained.

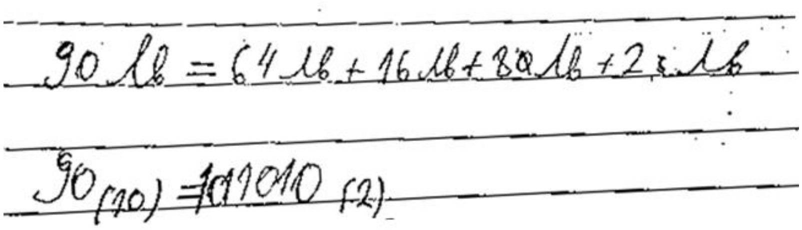

Figure 3. *This excerpt from a student's notebook demonstrates both perfect understanding of the algorithm I explained and mental math calculations of the money residuals*

3. An unexpected mathematical challenge

If a professional mathematician had created the story-problem discussed, then he would have probably supplied 1,023 coins to be distributed among the ten empty purses[3]: 1 coin into the first purse, 2 into the second, 4 into the third, etc., up to 512 coins into the tenth. This was also the hint provided to the readers of *Path* two issues later. However, neither the editors nor the readers noticed the discrepancy between the number of 1,023 coins needed for this distribution and the only 1,000 coins available.

Decades later one can only speculate why that amount of money had been chosen as numerical data. For the ordinary Bulgarian people who lived in the period between the Two World Wars, 1,000 levs was a huge amount of money. Therefore the editors of the newspaper should have thought that this money would be enough to fill the purses according to the hint. While for math professors it should have been elementary truth that the sum 1 + 2 + 4 + … + 512 was 1,023, the publishers were probably not aware of it. However, this number made the situation even more valuable from a mathematical perspective and initiated an interesting in-class discussion about how the 1,000 levs were to be distributed among the 10 purses. Its outlines are graphically represented in *Figure 4*.

Using number 489 together with the powers of 2 ruins the binary number representation. The consequences of that are quite intriguing. Although any integer between 1 and 1,000 inclusive can be represented through this set of addends, the representations are not always unique. Since 489 = 256 + 128 + 64 + 32 + 8 + 1, the sum of 489 can be expressed through six addends or just through one. Since 511 is both equal to $2^0 + 2^1 + 2^2 + ... + 2^8$ and 489 + 16 + 4 + 2, twofold representations exist for all numbers between 489 and 511 inclusive. The numbers not greater than 488 are sums only of the powers 1, 2, 4, 8, … , 256, while these from 512 to 1,000 must contain 489 as an addend.

Figure 4. *Distributing 1,000 one-lev coins into 10 purses to pay any sum between 1 and 1,000 levs*

4. Writing the binary code

In his seminal work on foreign language education Butzkamm (2003) notes that using their mother tongue people learn to think, communicate, and understand grammar. Thus he infers that the mother tongue is "the greatest assest people bring to the task of foreign language learning". On the other hand, the great German poet and philosopher Johann Wolfgang von Goethe writes that "anyone who doesn't know foreign languages knows nothing of his own" (Goethe, 1821, p. 10).

In my opinion, these opposite to some extent statements are quite complementary. Although they concern a different area of human knowledge, they can be still applied to the learning of binary number system in primary school. My experience shows that the third graders absorbed the new idea of representing numbers by the powers of 2 through the analogy with the familiar decimal number system. The latter played the role of a mathematical "mother tongue" and supported their understanding of the binary number system while it seemed a "foreign language" to them.

Speaking, understanding, and writing in any language require grammar. Similarly, representing and operating with numbers in number systems also obeys rules specific for the nonpositional systems like Egyptian, Babylonian, Greek, Roman, etc. and the positional ones like binary, ternary, decimal, hexadecimal, etc. (Eves, 1983, pp. 4−16).

By distributing the 1,000 coins among the ten empty purses the third graders answered the story-problem question. However, the analogies between the decimal and binary systems I used kept their interest. Being aware that only the digits of 0 and 1 were allowed, the pupils tried to write several numbers in a binary code. Unfortunately, their efforts not grounded in the algorithm were unsuccessful.

According to the Bulgarian curriculum, division with remainders is studied in the middle school and teaching the standard algorithm for convertion from decimal to binary representation (*Table 1*) will not be based on the students' prior mathematical knowledge.

Table 1. *Based on writing backwards the remainders of successive divisions by 2, the standard algorithm for converting numbers from decimal to binary representation was not accessible to the third graders*

Dividend	**90** : 2 =	**45** : 2 =	**22** : 2 =	**11** : 2 =	**5** : 2 =	**2** : 2 =	**1**
Remainder	**0**	**1**	**0**	**1**	**1**	**0**	

Since the matter is how the sequence of zeros and ones is to be obtained, this shortage can be avoided. The primary school students distinguish even from odd numbers and can perform all the divisions by 2, each time marking the even dividends by 0s and the odd – by 1s. As part of the process, the last dividend, which is always 1, is to be marked by 1.

Although accessible to the third grade students, this explanation of the algorithm still contains an obstacle. My observations show that even undergraduate students tend to forget whether the binary code is to be composed by backward or forward re-writing of the sequence of 0s and 1s. Therefore, the primary school pupils needed another approach to the Base-10 to Base-2 conversion which would not perplex them at the final stage.

As previously mentioned, the pupils found my method for choosing the right purses easy and entertaining. Curious what some specific numbers would look like, they set themselves the challenge of writing down the binary code of number 90. They asked for instructions and I tried to offer them an easy way to deal with that.

My approach continued the idea of weights associated with the numbers which the class had already applied. In binary system where the numbers are represented as sums of successive powers of 2 with coefficients 0 or 1, the coefficients of 1 denote that the respective powers of 2 add to the sum, while the coefficients of 0 mean that they don't. Thus to write the binary representation of number 90, first the pupils were to imagine that a payment of 90 coins was to be paid and the purses of 1, 2, 4, 8, 16, 32, 64, 128, … coins were available. The purses with 128 coins or more did not come to use, therefore the students had 7 purses to choose among. This meant that the binary representation of number 90 had 7 digits. The leading digit designated that the purse with 64 coins had to be used first. It was followed by 0, because the purse with 32 coins had been omitted. Then two more digits of 1 were to be written to mark the purses with 16 and 8 coins used. The omitted purse of 4 coins was marked by the digit of 0, followed by the digit of 1 for the last purse of 2 coins used. At last the digit of 0 was added to designate that the purse of 1 coin was not used (*Figure 3*).

Figure 5. *Binary number system on an improvised stage: the classroom floor*

The finishing touch of pupils' activities was to show on stage what they had learned about binary number system. The improvised design of the purses focused on their contents and brought a lot of fun in the classroom. Part of the students were on stage, the other part were the audience. The audience gave the numbers to be converted and the actors did the task; then they changed their places and did more convertions so no student remained neglected. Thus the whole class demonstrated the new mathematical knowledge acquired (*Figure 5*).

Conclusion

Curriculum designers may argue if teaching the computer science related topic of binary number system is appropriate for primary school students. Interestingly, the distinguished scientist and engineer of Bulgarian ancestry John Atanassoff [4] was almost the age of these pupils when he learned from his mother, a mathematics schoolteacher, about binary number system. The early attained knowledge probably opened his mind to the advantages of representing numbers by sequences of 0s and 1s and operating with them in a revolutionary new machine: the Atanassoff-Berry computer. The years to come proved that the binary number system promoted in a 1930s children's newspaper was not just a link between manual and computer arithmetic. It was a bridge to a new era for humanity: the one of digital devices and information technologies.

Modern primary school pupils also deserve to be empowered by this mathematical knowledge in a way they love: through stories, fairy tales, or even mathematical dramas. The methods of teaching mathematics as storytelling are widely respected (Gerofsky, 1996; Zazkis and Liljedahl, 2009; Whitin and Whitin, 2004; etc.). The world of words reveals its magic and power to the pupils, nurtures their imagination and creativity, and conveys rich social and cultural messages from the times they were written. Thus story problems can build positive attitude to mathematics and problem solving (Gortcheva, 2010; Marchiş, 2013a; Marchiş, 2013b; Gortcheva, 2014) even though they were not always among the pupils' favorites.

The biggest reward from my teaching of binary number system came unexpectedly a year later. One day I just stopped by the class to say "Hi" to the already fourth graders. They cheerfully surrounded me and one of them asked: "Professor, when are we going to study university math again?" Apparently our activities had had a great impact on their motivation and desire to learn math.

Notes

[1] The children newspaper *Path* was issued in Sofia in the period 1932-1936 by Bulgarian writers Elin Pelin, Dimitar Podvarzachov, and Iordan Slivopolski-Piligrim. The mathematical story-problems published there are now collected in a separate book (Elin Pelin and Podvarzachov, 1992).

[2] Lev is the name of the Bulgarian currency.

[3] On page 10 of his book *Number theory* George Andrews (Andrews, 1994) provides the following problem: *What is the least number of weights required to weigh any integral number of pounds up to 63 pounds if one is allowed to put weights in only one pan of a balance?*

[4] John Vincent Atanassoff (1903-1995). Iowa State University, Ames Laboratory and US Department of Energy archive.

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